**System Noise-Figure Analysis for Modern Radio Receivers: Part 2, Y-Factor Noise-Factor Measurement**

By: **Charles Razzell**, Executive Director, Maxim Integrated

This article is the second half of a detailed discussion of noise factor for modern RF radio receivers. In Part 1 we discussed the general concept of noise figure and how it is used to convey noise-performance requirements by product definers and circuit designers. It is also used to predict the overall sensitivity of receiver systems. We also presented calculations for a cascaded receiver. In this continuation article we focus on the Y-factor measurement as it applies to mixers. We state which measurement is applicable to the cascade equations derived in Part 1. We also explore some variations of the measurement method that could be used to obtain an approximation to the SSB noise figure of a mixer.

**Noise Temperature**

To discuss Y-factor noise measurement, it is necessary to introduce the concept of noise temperature. In previous equations, we used the well-known result that noise-power spectral density available from a resistor at a given temperature is \( kT \), Watts/Hz, where \( k \) is Boltzmann’s constant and \( T \) is the absolute temperature. This makes it possible to account for all sources of noise in a device, if we pretend that the device is noiseless and if the additional noise-power spectral density is accounted for by an equivalent rise in noise temperature of the input termination above the reference temperature. The noise factor can be related to the equivalent temperature, \( T_e \), by \( F = 1 + T_e / T_0 \), where \( T_0 \) is defined as the reference noise temperature of 290K. Unsurprisingly, a noise factor of 1 is represented by an equivalent noise temperature of the device of 0K, whereas a noise factor of 2 is represented by \( T_e = 290K \).

**Y-Factor**

The Y-factor method of noise figure measurement involves the use of a calibrated noise source, which has two distinct noise temperatures depending on the presence or absence of DC power to the device. The calibrated source has a characterized excess noise ratio (ENR) defined as:

\[
ENR_{dB} = 10 \log_{10} \left( \frac{T_{S^{ON}} - T_{S^{OFF}}}{T_0} \right)
\]

Where \( T_{S^{ON}} \) is the noise temperature of the source in its ON state and \( T_{S^{OFF}} \) is the corresponding value in the OFF state. The Y-factor is a ratio of two noise power levels, one measured with the noise source ON and the other with the noise source OFF:

\[
Y = \frac{N^{ON}}{N^{OFF}}
\]

Since the noise power available from a source can be represented directly by its noise temperature, we can also write:

\[
Y = \frac{T^{ON}}{T^{OFF}}
\]

**Noise-Factor Measurement and Calculation**

To assess the noise factor of a device under test (DUT), we must connect a noise-power measurement device to the output of the DUT. Let the DUT have a noise temperature \( T_1 \) and the instrument have a noise temperature \( T_2 \). While it is impossible to eliminate the measurement device’s noise temperature \( (T_2) \) from any given reading, we can measure \( T_{12} \) which is the combined noise temperature of the DUT followed by the instrument. We can use calculations to isolate \( T_1 \) since \( T_{12} = T_1 + T_2 / G \). So, the strategy is to take a Y-factor measurement with the calibrated noise source connected directly to the measuring instrument which will allow \( T_2 \) to be determined. We have:
\[ Y_2 = \frac{N_{2_{ON}}^{ON}}{N_{2_{OFF}}^{OFF}} = \frac{T_{2_{ON}}^{ON} + T_2}{(T_{2_{OFF}}^{OFF} + T_2)} \]

Which can be rearranged as:

\[ T_2 = \frac{T_{2_{ON}}^{ON} - Y_2 T_{2_{OFF}}^{OFF}}{Y_2 - 1} \]

Having obtained the noise temperature of the measuring device, based on known values of \( T_{S_{ON}}^{ON} \) and \( T_{S_{OFF}}^{OFF} \), the next step is to measure a new Y-factor for the cascade of the DUT and the measuring instrument:

\[ Y_{12} = \frac{N_{1_{ON}}^{ON}}{N_{1_{OFF}}^{OFF}} \]

This allows the combined noise temperature of the DUT and the instrument to be calculated using the same procedure as before:

\[ T_{12} = \frac{(T_{S_{ON}}^{ON} - Y_{12} T_{S_{OFF}}^{OFF})}{(Y_{12} - 1)} \]

Having previously stored both \( N_{1_{ON}}^{ON} \) and \( N_{1_{OFF}}^{OFF} \) and now having access to \( N_{12_{ON}}^{ON} \) and \( N_{12_{OFF}}^{OFF} \), we have sufficient information to calculation the gain of the DUT as:

\[ G_1 = \frac{(N_{1_{ON}}^{ON} - N_{1_{OFF}}^{OFF})}{(N_{2_{ON}}^{ON} - N_{2_{OFF}}^{OFF})} \]

This provides sufficient information to mathematically subtract the contribution of the measuring instrument’s noise temperature, using:

\[ T_1 = T_{12} - T_2/G_1 \]

**Losses Before the DUT**

If there are known losses before the DUT, the impact of these losses must be removed to obtain the true noise temperature of the DUT at its input \( T_{1_{IN}}^{ON} \). Assuming that these losses are absorbative, the following equation can be used:

\[ T_{1_{IN}}^{ON} = (T_1/L_{IN}) - ((L_{IN} - 1)T_2/L_{IN}) \]

Where \( T_L \) is the physical temperature of the loss; and \( L_{IN} \) is the insertion loss to be compensated, and is expressed as a linear power ratio greater than unity.

**Mixer as DUT in Y-Factor Noise-Factor Determination**

Considering that the calibrated noise sources used for noise-figure measurement are broadband in nature and that any slight variation in noise temperature when ON is handled by a detailed calibration table embedded in the source, any unmodified use of the Y-factor technique will result in the evaluation of the DSB noise figure of the mixer. This is because the calibrated noise source will inject noise power in both sidebands simulataneously, and the combined output noise power from both sidebands will contribute to the output noise temperatures used to calculate the Y-factor.

**Example of DSB Noise-Figure Measurement by Y-Factor Method**

To illustrate the concepts discussed, a Genesys simulation is performed by injecting a noise source into the simulated DUT, which is a mixer with a DSB noise figure of 4.9dB and a conversion gain of 8.8dB. The noise power injected is determined by the variable PIN, which is a swept variable that iterates over two possible values, -159dBm/Hz and -174dBm/Hz, representing the ON and OFF conditions of the noise source, respectively. The IF frequency is defined to be 250MHz with wanted and image responses at 2000MHz and 1500MHz in the RF port of the mixer (Figure 1). The only data collected by the simulation (Tables 1 and 2) are the channel noise power in 100kHz bandwidth both at the input (i.e., directly connected to the noise source in lieu of a calibration step) and at the output (representing the measurement mode).
Figure 1. Simulation schematic to determine DSB mixer noise figure using Y-factor.

<table>
<thead>
<tr>
<th>B (Hz)</th>
<th>IL (dB)*</th>
<th>pinOFF (dBm)</th>
<th>pinON (dBm)</th>
<th>poutOFF (dBm)</th>
<th>poutON (dBm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100,000</td>
<td>0</td>
<td>-123.975</td>
<td>-109</td>
<td>-107.265</td>
<td>-96.91</td>
</tr>
</tbody>
</table>

*Note that the parameter IL represents the insertion loss before the DUT, which is 0dB in this case.

Table 2. Y-Factor Calculations for DSB Mixer Measurement

<table>
<thead>
<tr>
<th>Y_2</th>
<th>Y_12</th>
<th>T_12 (K)</th>
<th>T_2 (K)</th>
<th>T_1 (K)</th>
<th>T_1^IN (K)</th>
<th>F (dB)</th>
<th>G (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>31.443</td>
<td>10.851</td>
<td>606.147</td>
<td>0</td>
<td>606.147</td>
<td>606.147</td>
<td>4.9</td>
<td>8.8</td>
</tr>
</tbody>
</table>

Note that T_2 represents the noise temperature of the instrument which is acceptable, since the instrument is, in this case, the Genesys simulator which evaluates noise without adding any of its own. Because the insertion loss before the DUT is 0dB, T_1 is identical to T_1^IN. The final calculated noise figure from the Y-factor measurements is given by \( F = 10 \log_{10}(1 + T_1^{IN}/290) \) and the value obtained (4.9dB) aligns with the expected value from the parameter settings used when setting up the mixer schematic.

Example of SSB Noise-Figure Measurement by Y-Factor Method

The simulation schematic is shown in Figure 2 and the test results in Tables 3 and 4.

Figure 2. Simulation schematic to determine SSB mixer noise figure using Y-factor.

<table>
<thead>
<tr>
<th>B (Hz)</th>
<th>IL (dB)*</th>
<th>pinOFF (dBm)</th>
<th>pinON (dBm)</th>
<th>poutOFF (dBm)</th>
<th>poutON (dBm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100,000</td>
<td>2.2</td>
<td>-123.975</td>
<td>-109</td>
<td>-108.015</td>
<td>-101.455</td>
</tr>
</tbody>
</table>

*Note that the parameter IL represents the insertion loss before the DUT, which is 2.2dB in this case.
Table 4. Y-Factor Calculations for SSB Mixer NF Measurement

<table>
<thead>
<tr>
<th>(Y_2)</th>
<th>(Y_{12})</th>
<th>(T_{12}) (K)</th>
<th>(T_2) (K)</th>
<th>(T_1) (K)</th>
<th>(T_{1IN}) (K)</th>
<th>(F) (dB)</th>
<th>(G) (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>31.443</td>
<td>4.529</td>
<td>2211.584</td>
<td>0</td>
<td>2211.584</td>
<td>1217.354</td>
<td>7.158</td>
<td>6.602</td>
</tr>
</tbody>
</table>

Because the insertion loss before the DUT is 2.2dB, \(T_1\) is higher than the mixer’s noise temperature to \(T_{1IN}\), which has been calculated according to the equation in the section Losses Before the DUT above. The final calculated noise figure from the Y-factor measurements is given by \(F = 10\log_{10}(1 + T_{1IN}/290)\) and the value obtained is 7.158dB. This should be compared to the value obtained, assuming that the image noise of the source is completely suppressed:

\[NF = 10\log_{10}(2(10^{(4.9/10)} - 1) + 1) = 7.144\, dB\]

Since the filter has finite insertion loss, the impedance of the image suppression filter at the image frequency is not entirely reactive. This, in turn, implies that the image-band noise of the source is not fully suppressed. This is assumed to be the cause of the small increase from the ideal noise figure.

Example of SSB Noise-Figure Measurement by Padded Y-Factor Method

In this method we apply an attenuator to ensure that the mixer is subject to similar “cold” (i.e., OFF-state) noise temperatures at both the wanted and image frequencies. This should result in the SSB noise figure more closely approximating a value 3dB higher than the DSB noise figure, since the noise temperature of the source termination is no longer colored by the filter to any significant extent (Figure 3, Tables 5 and 6).

![Figure 3. Simulation schematic to determine SSB mixer noise figure using padded Y-factor.](image)

Table 5. Y-Factor Simulation Results for Padded SSB Mixer Measurement

<table>
<thead>
<tr>
<th>(B) (Hz)</th>
<th>(IL) (dB)*</th>
<th>(pinOFF) (dBm)</th>
<th>(pinON) (dBm)</th>
<th>(poutOFF) (dBm)</th>
<th>(poutON) (dBm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100,000</td>
<td>12.2</td>
<td>-123.975</td>
<td>-109</td>
<td>-107.272</td>
<td>-106.141</td>
</tr>
</tbody>
</table>

*Note that the parameter IL represents the insertion loss before the DUT, which is 12.2dB in this case to represent the combined loss of the filter and attenuator.
Table 6. Y-Factor Calculations for Padded SSB Mixer NF Measurement

<table>
<thead>
<tr>
<th>$Y_2$</th>
<th>$Y_{12}$</th>
<th>$T_{12}$ (K)</th>
<th>$T_2$ (K)</th>
<th>$T_1$ (K)</th>
<th>$T_{1\text{IN}}$ (K)</th>
<th>$F$ (dB)</th>
<th>$G$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>31.443</td>
<td>1.297</td>
<td>29392.313</td>
<td>-5.98E-14</td>
<td>29392.313</td>
<td>1498.536</td>
<td>7.901</td>
<td>-3.398</td>
</tr>
</tbody>
</table>

The final calculated noise figure from the Y-factor measurements is given by $F = 10\log_{10}(1 + T_{1\text{IN}}/290)$ and the value obtained is 7.901dB. This corresponds well to the value that would be anticipated from adding 3.0dB to the DSB noise figure of 4.9dB. Note that the use of a 10dB attenuator causes the Y-factor to get close to unity, which may endanger accuracy. When using high values of attenuation in a real-world measurement, it is advisable to select the highest ENR source available to maintain accuracy.

**Y-Factor Conclusions**

The Y-factor measurement will evaluate the DSB noise figure of a mixer unless special actions are taken to filter out the broadband noise stimulus at the image frequency. This is the appropriate value to be used with the cascade equations derived in Part 1 of this article. When a filter is used in an attempt to obtain the SSB noise figure, it is necessary to account for the insertion loss of the filter used. Furthermore, the degree of source termination image-noise suppression caused by the filter can cause a deviation from the classical definition of SSB noise figure. Use of a matched attenuator can overcome this problem to a large extent, providing that the amount of attenuation used is not excessive compared to the ENR of the noise source.

**Overall Conclusions**

For RF system engineers, calculations of a noise-figure budget are critically important to predicting product performance. Confusion has typically arisen when accounting for image noise in the definition and calculation of noise figures for mixers. The common understanding that the SSB NF of mixers is 3dB higher than the corresponding DSB noise figure is not always true—it depends on the conversion gain at both image and wanted frequencies being equal. Analysis has shown that a naive application of the Friis cascaded noise equation can be a source of error; in particular, zero-IF and low-IF receiver architectures need to be handled with specific variations of that equation, which we derived. Finally, the Y-factor method of noise measurement was described and its application to mixer noise-figure measurement was discussed.

**Reference**


About the Author:

**Charles Razzell** received his undergraduate electronics engineering degree from the University of Manchester Institute of Science and Technology (UMIST), U.K. Since then, he has been involved in development and advanced development projects, usually with a focus on highly integrated transceiver designs. During the 1980s his technical work was mainly in the analog and RF design fields, including the design and layout of one of the earliest fully-integrated paging receiver ICs for Philips. His involvement in digital radio technology began in the early 1990s with TErrestrial Trunked Radio (TETRA), where he was involved in making a system proposal on behalf of Philips and made further contributions to the Physical Layer specification of that ETSI standard. At Maxim Integrated he is an Executive Director with responsibility for product definition, applications and product engineering. He has eighteen granted U.S. patents.