MWJ Puzzler

Math, Science and Logic Puzzles for the 'Enginerd' in All of Us

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Thousand Monkeys

A big building in which a thousand monkeys are living is lit by a thousand lamps. Every lamp is connected to a unique on/off switch, that are numbered from 1 to 1000. At some moment, all lamps are switched off. But because it is becoming darker, the monkeys would like to switch on the lights. They will do this in the following way.

Monkey 1 presses all switches that are a multiple of 1.
Monkey 2 presses all switches that are a multiple of 2.
Monkey 3 presses all switches that are a multiple of 3.
Monkey 4 presses all switches that are a multiple of 4.
Etc.
How many lamps are switched on after monkey 1000 pressed his switches?
And which lamps are switched on?

Solution

To figure out the state of lamp number 10 after all monkeys have pressed their buttons: 10 = 2*5, 10 = 5*2, 10 = 10*1 and 10 = 1*10. So 10 is a multiple of 1, 2, 5 and 10. Hence monkey 1, monkey 2, monkey 5 and monkey 10 have pressed on switch 10. This implies that lamp 10 is switched off when all monkeys have pressed their buttons.

So if we take a lamp number x. Of course it holds that x = 1 * x and x = x * 1. x is always a multiple of 1 and of x. If x is a prime, then there are no other more possibilities, so that the lamp is switched off at the end. If x is not a prime there is at least one way in which we can write x = n * m, where n and m are integers. So x is a multiple of n and of m, hence monkey n and monkey m press button x. If x is not a square, x has an even number of different multiples. Then x will be switched off at the end. But if x is a square, x = n*n, in other words x = 1, 4, 9, ... then x has an odd number of multiples. That implies that if x is a square, lamp x will be on at the end.

Since 31*31=961 and 32*32 = 1024 there are 31 different squares below 1000. Hence, 31 lamps are switched on at the end, these are 1, 4, 9, 16, 25, ... and 961.

Beach Walk

Along the beach are a number of poles standing at equal distance from each other. The poles are numbered 1, 2, 3, 4, ... Alice is walking from the first pole to the last one and back, Bob is doing that in the opposite direction. They start at the same time and walk with constant, but different velocities. Their first encounter is at pole number 10, their second (when they're both on the way back) at pole number 20. How many poles are standing along the beach?

Solution

The distance between the poles is not important; assume for example it is 100 m. Let us call the distance between the first and the last pole D. At the first encounter, Alice and Bob have together walked a distance D. At the second encounter (as one can see by making a sketch) they have together walked 3 times as much, so 3D. At the first encounter Alice has walked 900 m, so at the second encounter she has walked 2700 m. Then she still has to go back 1900 m to her start at pole 1. So in total Alice walked 4600 m. From this it follows that D = 2300 m, so there are 24 poles along the beach.

This puzzle can also be solved using equations. Let us call the velocity of Alice v_a and that of Bob v_b and the number of poles x. At the first encounter at time t_1 it holds that

$t_1 = 9 / v_a = (x - 10) / v_b$.

At the second encounter at time t_2 it holds that

$t_2 = (x - 1) / v_a + (x - 20) / v_a = (x - 1) / v_b + 19 / v_b$.

These equations can be rewritten into

$9 v_b = (x - 10) v_a$

$(2x - 21) v_b = (x + 18) v_a$

If we divide the upper equations by each other and multiply by the denominators we obtain the following quadratic equation

$x^2 - 25 x + 24 = 0$.

This equation has as solution x = 1 and x = 24. x = 1 is not a correct solution, since there are at least 20 poles along the beach. So we can conclude that there are 24 poles along the beach.

Also Having Birthday Today?

What is the smallest size that a group of people can be in order to have that the probability that two people out of the group have a birthday on the same day be larger than 1/2?

Solution

Surprisingly, we only need 23 people in order to assure that the probability that two people have a birthday on the same day is larger than 1/2.

The probability that at least two people have a birthday on the same day is 1 minus the probability that everybody is having a birthday on different day. The probability that everybody out of a group with x people is having a birthday on a different day is equal to $P(x) = [365*364*363*....*(366-x)]/(365^x)$. x = 23 is the smallest number for which $1 - P(X) > 1/2$. This proves that the group should contain at least 23 people in order to have a probability that at least two people are having birthday on the same day is larger than 1/2.

Just Adding

How much is

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\dots}}}}$$

Solution

Let us call the number we search for x. It holds that $x = 1 + 1 / x$. It now follows that $x^2 - x - 1 = 0$. Solving this equation gives

$$x = \frac{1+\sqrt{5}}{2} = 1.61803...$$

This number is also called the golden ratio.