UNDERSTANDING OFFSET 8-PSK MODULATION FOR GSM EDGE

The evolution of the GSM standard for mobile communications toward higher data rates embodied by the enhanced data rates for GSM evolution (EDGE) uses a variation of an eight-phase-shift keying (8-PSK) modulation scheme. Understanding of the selected modulation technique and the reasons behind it is important because they drive design choices. This article describes 8-PSK and its derivation toward the form it takes in GSM EDGE. A complete GSM EDGE transmitter/receiver baseband architecture is discussed along with simulation results.

The need to expand mobile communications systems capabilities to encompass data as well as voice means that higher data rates will be transmitted without loosening bandwidth restrictions. This enhancement requires the use of spectrally more efficient modulation techniques, that is, modulation techniques that offer a higher throughput/occupied bandwidth ratio. An evolution of the current GSM standard to provide higher data rates is GSM EDGE (812.5 kbps) where the modulation scheme to be used is pulse-shaped (Gaussian) 3π/8 offset 8-PSK as opposed to GSM (270.8 kbps), which uses Gaussian minimum-shift keying (GMSK). One of the most remarkable differences between the two modulation formats is that GMSK has a constant amplitude or envelope and exhibits phase modulation; pulse-shaped 8-PSK exhibits both amplitude and phase variations.

8-PSK

Analog or digital modulation consists of varying the characteristics of a sinusoidal waveform u(t) (the carrier), be it amplitude, phase, frequency, polarization or a combination of these characteristics, according to the information being transmitted. In the case of digital modulation schemes, the information is in a digital format, usually a binary word (n bits), hence producing $M = 2^n$ possible words. Each symbol has a duration T and each bit has a duration T_b, where $T = nT_b$.

M-PSK is a digital modulation scheme where the information to be transmitted is conveyed onto the phase of the carrier. The expression for the modulated carrier u(t) is

$$u(t) = A(t)\cos\left(2\pi f(t)t + \phi(t)\right) \qquad (1)$$

and the phase of the carrier φ(t) is

$$\phi(t) = \sum_k \phi_k \delta(t - kT) \qquad (2)$$

where

$$\phi_k = \theta_0 + (2m+1)\frac{\pi}{m}$$
$$m \in [0 \rightarrow M-1]$$

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ϕ_k is a set of M-ary words and $\delta(t)$ is a Dirac pulse. Therefore, the expression for the modulated carrier can be written as

$$\begin{aligned} u(t) &= A(t)\cos\left(2\pi f(t)t + \sum_k \phi_k \delta(t-kT)\right) \\ &= \sum_k A(t)\cos\left(2\pi f(t)t + \phi_k\right)\delta(t-kT) \\ &= \sum_k A\cos\left(2\pi ft + \phi_k\right)\delta(t-kT) \qquad (3) \end{aligned}$$

Since amplitude and phase are constant in M-PSK, this expression can be expanded as

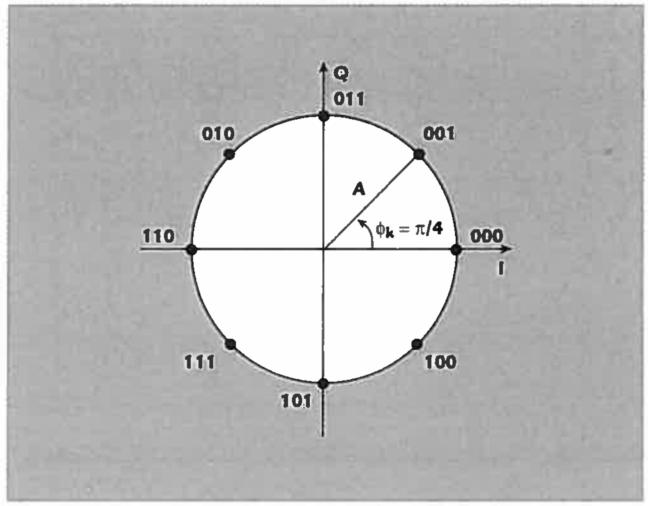
$$\begin{aligned} u(t) &= A\sum_k\left[\cos(\phi_k)\cos(2\pi f_c t) - \sin(\phi_k)\sin(2\pi f_c t)\right] \\ &\quad \bullet\delta(t-kT) \\ &= \sum_k\left[I_k\cos(2\pi f_c t) - Q_k\sin(2\pi f_c t)\right]\delta(t-kT) \qquad (4) \end{aligned}$$

where

$I_k = A\cos(\phi_k)$
$Q_k = A\sin(\phi_k)$
f_c = carrier frequency

Traditionally, I_k and Q_k are referred to as the in-phase and quadrature components since u(t) can be decomposed as a sum of two quadrature waveforms $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$. These components take their values in an M-ary alphabet formed by n bits ($M = 2^n$). This expression can be represented in the Fresnel plane (complex plane referring to a complex envelope representation of the signal).

For 8-PSK, $M = 8 = 2^3$ where n = 3. Therefore, each symbol is formed from three bits. A vectoral illustration of this technique is shown in ***Figure 1***, with $\theta_0 = 0$ and a Gray coding for mapping the bits to the symbols.



▲ *Fig. 1 An 8-PSK constellation diagram with Gray coding.*

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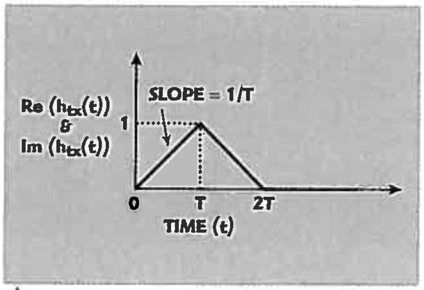
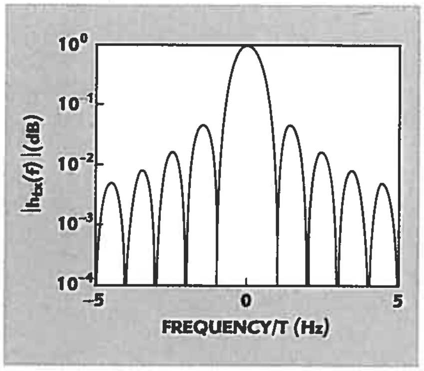
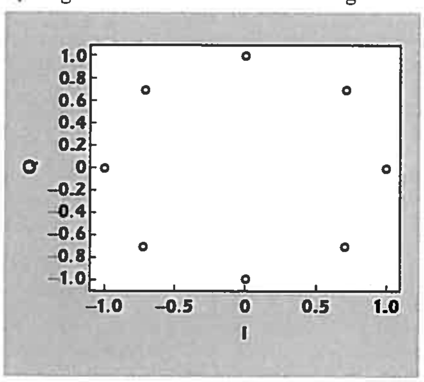


▲ *Fig. 2 The triangular filter's impulse response.*

Fig. 3 The triangular filter's frequency response. ▼



▼ *Fig. 4 An 8-PSK constellation diagram.*



PULSE SHAPING

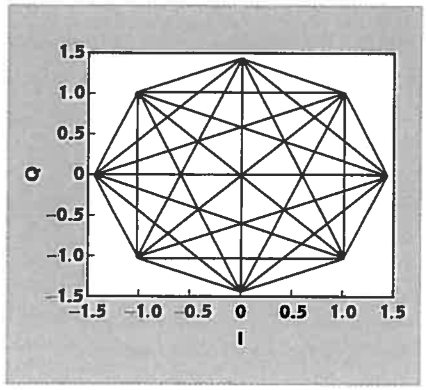
Considering the complex envelope of the signal $u_e(t)$ given by

$$u_e(t) = \sum_k \exp(j\phi_k)\delta(t - kT) \quad (5)$$

the spectrum of this signal is represented as

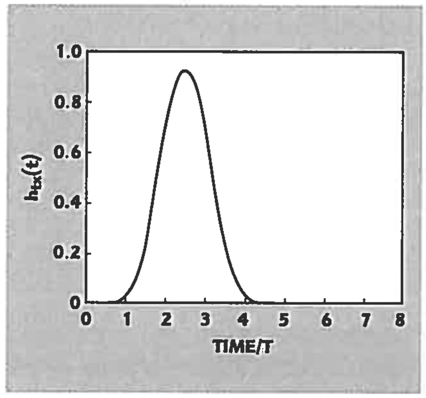
$$FT(u_e(t)) = U_e(f) = \sum_k \exp(j(\phi_k + 2\pi fk)) \quad (6)$$

Since ϕ_k has a random distribution, $U_e(f)$ also has a random distribution and can be regarded as having an infinite bandwidth. In practice, it is not tolerable to transmit such a signal since only a limited bandwidth is available. Therefore, the signal needs



▲ *Fig. 5 Trajectories for the triangular filter.*

Fig. 6 The GSM EDGE pulse-shaping filter's impulse response. ▼



to be filtered in order to limit its spectrum. A useful filter is a triangular filter for which the time and frequency domain responses are shown in ***Figures 2*** and ***3***, respectively. (This filter can be derived easily.)

It is often mistakenly thought that 8-PSK signals' trajectories are linear transitions between one constellation point and the next. In fact, an 8-PSK signal as represented by its I and Q components has no trajectory as such because instantaneous phase changes occur between symbols at every T. ***Figure 4*** shows an 8-PSK constellation diagram. This filter produces linear monotonic trajectories, as shown in ***Figure 5***, from one symbol to the following symbol and it can be seen from its impulse response that the output depends on two consecutive symbols. (T is the period of a symbol.) The frequency response of the filter displays a reduced spectrum (–3 dB BW ≈ 0.32/T Hz).

The proposed filter for GSM EDGE is a Gaussian-like filter for which the impulse and frequency response are shown in ***Figures 6*** and

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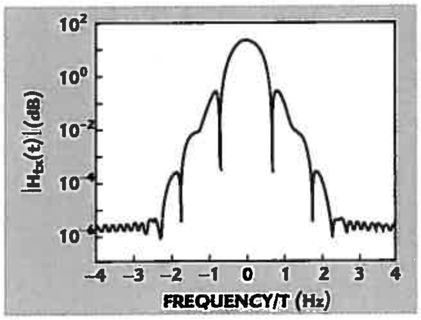
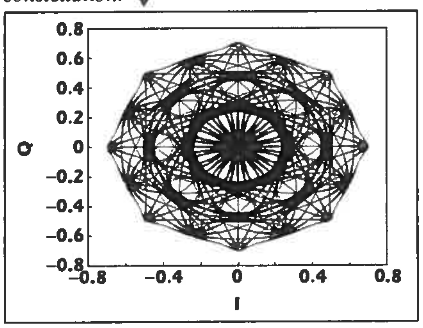
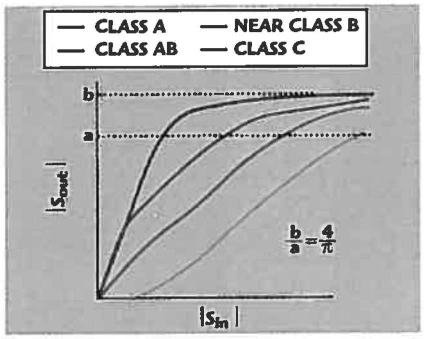


▲ *Fig. 7 The GSM EDGE pulse-shaping filter's frequency response.*

Fig. 8 The filtered GSM EDGE 8-PSK constellation. ▼



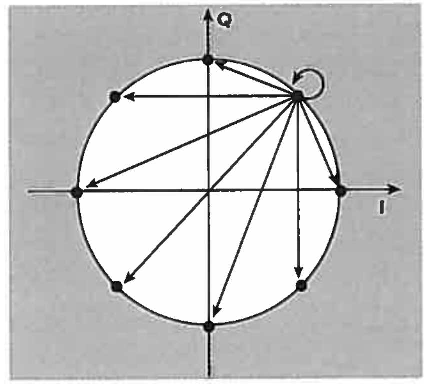
7, respectively. (The impulse response of the filter as it is defined in the European Telecommunications Standards Institute (ETSI) standards is discussed in ***Appendix A***.) It can be seen that the output of this filter is dependent on five consecutive symbols. The trajectory of the filtered I and Q signals is shown in ***Figure 8***. It is worth noting that the nonfiltered I and Q signals are of a constant amplitude although the filtered signal undergoes both phase and amplitude variations.



▲ *Fig. 9 Typical PA characteristics.*

OFFSET PULSE-SHAPED 8-PSK

In both of the examples discussed earlier, the trajectory of the modulating data (I and Q) goes through the origin (I = Q = 0). In practice, power amplifiers exhibit a nonlinear characteristic not only for high power regions but also for low power regions, as shown in ***Figure 9***. This condition is especially true for class C amplifiers where the base-emitter junction threshold is required to start conducting before the amplifier kicks in. In other words, for modulation schemes such as 8-PSK, where the trajectory crosses the origin, nonlinearity at low power regions will distort the signal and, hence, produce spectral regrowth. ***Figure 10*** shows all



▲ *Fig. 10 All possible 8-PSK transitions.*

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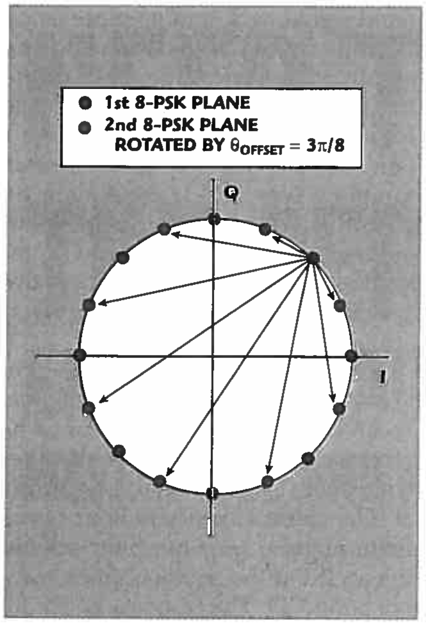
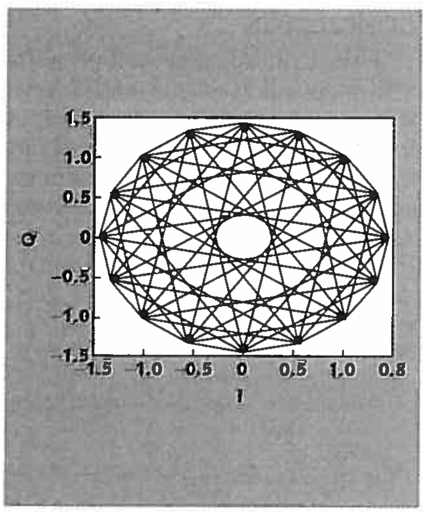


▲ *Fig. 11 Offset 8-PSK transitions.*

Fig. 12 The pulse-shaped (triangular) 3π/8 offset 8-PSK constellation. ▼

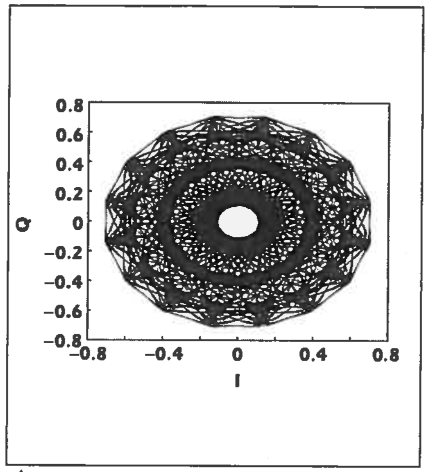


of the possible transitions from one symbol to the next.

It would be useful if the origin somehow could be avoided for the modulating data. One way of doing this is to offset the modulation scheme. In 8-PSK, the symbol at present time kT is $e(kT) = \exp(j\phi_k)$ and at time $(k + 1)T$ is $e((k + 1)T) = \exp(j\phi_{k+1})$, with $\phi_{k+1} \equiv \phi_k \bmod[\pi/4]$.

Continuously rotating the symbols by an offset $\theta_{offset} \equiv \pi/8 \bmod[\pi/4]$ prevents the signal from crossing through the origin. Hence, the transmitted signal becomes

$$u_{e\_offset}(t) = \sum_k \exp\left(j\left(\phi_k + k\theta_{offset}\right)\right)\delta\left(t - kT\right) \quad (7)$$



▲ *Fig. 13 The pulse-shaped (GSM EDGE) 3π/8 offset 8-PSK constellation.*

This condition is viewed as having two 8-PSK constellation planes offset by θ_{offset} and swapping from one plane to the other at every consecutive symbol time as shown in ***Figure 11***, where $\theta_{offset} = 3\pi/8$. This procedure can be illustrated using the triangular filter mentioned earlier and filtering the I and Q signals, where the benefit of rotating two 8-PSK planes leads to avoiding the center region of the constellation, as shown in ***Figure 12***. Applying the offset to the 8-PSK constellation and filtering it with the GSM EDGE pulse-shaping filter produces the output constellation shown in ***Figure 13***. Again, it can be seen that the trajectories no longer cross the origin.

One disadvantage of this procedure is that it may put strain on the detection for which the threshold margins are reduced. However, since there is a change in planes from one symbol to the next, the decision-making process in the receiver approximates that of an 8-PSK signal rather than, for example, a 16-PSK signal.

RECEIVER FILTER

In general, the baseband signal described previously is subsequently upconverted, transmitted via the transmission channel and demodulated in the receiver where a baseband signal is recovered. However, this example assumes that there is no up- or downconversion and that the transmission channel is ideal. In other words, to keep things simple, the analysis is performed at baseband according to the block diagram shown

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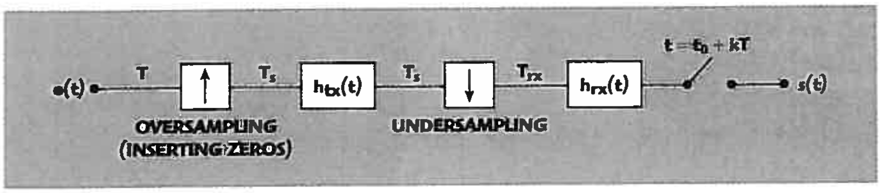
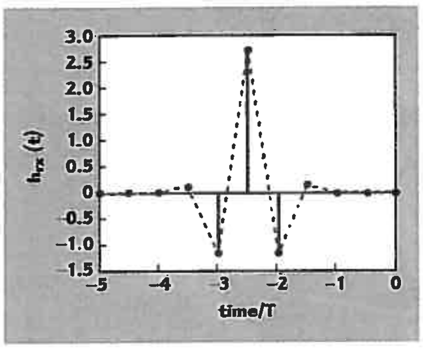
in **Figure 14**. (T = symbol period, T_s = sampling period (oversampling ⇒ $T_s \leq T$) and T_{rx} = sampling period going through the receiving filter. It is envisageable to have $T_s = T_{rx} = T_b$ (T_b = bit period) with appropriate zero padding.)

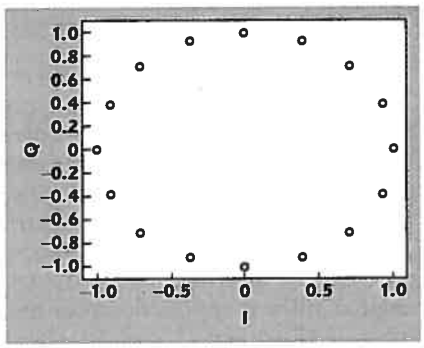
Although working at baseband, the transmitted signal, once it has been filtered (pulse-shaping filter $h_{tx}(t)$), is difficult to interpret. (Error vector magnitude can be used to assess the received signal.) Hence, it is desirable to convert the transmitted filtered signal back into a constellation as done initially, and then do the comparison. As can be seen in the transmitted signal, the main obstacle is that the signal at a given time t (and, more specifically, at the symbol sampling time kT) is dependent on several samples, or symbols (depending on the undersampling rate — recall the Gaussian filter spans over five symbol periods), referred to as intersymbol interference (ISI).



▲ *Fig. 14 The transmit/receive chain.*



▲ *Fig. 15 The zero-forcing GSM EDGE filter's impulse response.*



▲ *Fig. 16 The received constellation with no ISI.*

The aim here is to determine a receiver filter $h_{tx}(t)$ that will enable the received constellation diagram to be recovered. The derivation of the filter coefficients is explained in ***Appendix B***. The chosen filter here is an 11-tap finite impulse response filter exhibiting no ISI at the decision times (symbol period T). The response of the filter and the recovered constellation diagram are shown in ***Figures 15*** and ***16***, respectively.

CONCLUSION

This article has described an 8-PSK modulation scheme and some of its variations as well as its application to the GSM EDGE standard. It also explained a means of determining a complete baseband transmitter and receiver architecture. Additional information can be obtained from the author. ■

References

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2. John G. Proakis, *Digital Communications*, Third Edition, McGraw-Hill, New York, 1995.
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APPENDIX A

GSM EDGE pulse shaping filter impulse response h_{tx}

$$h_{tx}(t) = \begin{cases} \prod_{i=0}^{3} S(t+iT) & \text{for } 0 \le t \le 5T \\ 0 & \text{elsewhere} \end{cases}$$

where

$$S(t) = \begin{cases} \sin\left(\pi \int_0^t g(x)dx\right) & \text{for } 0 \le t \le 4T \\ \sin\left(\frac{\pi}{2} - \pi \int_0^{t-4T} g(x)dx\right) & \text{for } 4T \le t \le 8T \\ 0 & \text{elsewhere} \end{cases}$$

$$g(t) = \frac{1}{2T}\left(Q\left(2\pi \bullet 0.3 \frac{t - \frac{5T}{2}}{T\sqrt{\log_e(2)}}\right) - Q\left(2\pi \bullet 0.3 \frac{t - \frac{3T}{2}}{T\sqrt{\log_e(2)}}\right)\right)$$

and

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^{+\infty} \exp\left(\frac{-x^2}{2}\right)dx = \frac{1}{2}\,\mathrm{erfc}\left(\frac{t}{\sqrt{2}}\right)$$

for GSM EDGE, T = 3.69 µs

APPENDIX B

Derivation of the Receiver Filter h_{rx}

The input signal is the complex 3π/8 offset 8-PSK signal e(t)

$$e(t) = \sum_k \exp\left(j\left(\phi_k + k\frac{3\pi}{8}\right)\right)\delta(t-kT) \quad \text{(A1)}$$

This signal is then filtered by the Gaussian-like pulse shaping filter and filtered again by the receiver filter. The received signal s(t) can be written as

$$\begin{aligned} s(t) &= e(t) \otimes h_{tx}(t) \otimes h_{rx}(t) \\ &= e(t) \otimes g(t) \quad \text{with} \quad g(t) = h_{tx}(t) \otimes h_{rx}(t) \quad \text{(A2)} \end{aligned}$$

where ⊗ stands for convolution

To recover the transmitted signal (constellation), it is desirable to make S(kT) = e(kT)

$$s(kT) = e(kT)$$

$$s(kT_s) = \sum_n e(nT_s)g(kT_s - nT_s)$$

at $t = kT$:

$$\begin{aligned} s(kT) &= \sum_n e(nT_s)g(kT - nT_s) \\ &= e(kT)g(kT-kT) + \sum_{nT_s \ne kT} e(nT_s)g(kT - nT_s) \\ &= e(kT)g(0) + \sum_{n \ne k} e(nT)g(kT-nT) \\ &\quad \text{because } e(nT_s) = 0 \text{ for } nT_s \ne lT \\ &= e(kT)g(0) + \sum_{n,m \ne 0} e(nT)g(mT) \;\; \forall e(t) \quad \text{(A3)} \end{aligned}$$

Hence, the condition to be satisfied to guarantee no ISI is

$$\forall k \in Z^{\bullet} \begin{cases} g(0) = 1 \\ g(kT) = 0 \end{cases} \quad \text{(A4)}$$

Note that it is not necessary to define the behavior of g at all $t = nT_s$

$$\underline{g(0)=1} \Leftrightarrow g(0) = \sum_n h_{tx}(nT_s)h_{rx}(0 - nT_s)$$

$$\Leftrightarrow \sum_n h_{tx}(nT_s)h_{rx}(-nT_s) = 1 \qquad \Rightarrow \text{if } h_{tx} \text{ is causal then } h_{rx} \text{ is not causal}$$

$$\underline{g(kT)=1} \Leftrightarrow \sum_n h_{tx}(nT_s)h_{rx}(kT - nT_s) = 0 \qquad \text{for } k \ne 0$$

This system of linear equations leads to a matrix equation. If the receiver filter is defined every T/m step, $h_{rx}(t)$'s impulse response will be defined by a set of 2n + 1 coefficients b_{-2n}, b_{-2n+1}, …, b_0 and $k \in [-n \rightarrow +n]$. In other words, the value of g is specified at 2n + 1 points. This requires 2n + 1 samples of the transmitter pulse shaping filter $h_{tx}(t)$ (a_0, a_1, …, a_2) to be taken. Note that the receiver filter $h_{rx}(t)$ is noncausal here, hence the choice of negative indices. This type of filter is sometimes referred to as a zero-forcing filter. The set of equations can be written as

[Continued on page 92]

APPENDIX B (continued)

$$\begin{array}{r} k=-n \\ k=-n+1 \\ \\ \\ k=0 \\ \\ j \\ \\ \\ k=+n \end{array}\underbrace{\begin{bmatrix} a_0 & 0 & \cdots & & & & & & & 0 \\ a_{-m} & a_{-m+1} & \cdots & a_0 & 0 & \cdots & & & & 0 \\ a_{-2m} & a_{-2m+1} & \cdots & a_{-m} & a_{-m+1} & \cdots & a_0 & 0 & \cdots & 0 \\ \vdots & & & & & & & & & \\ a_{-2n} & a_{-2n+1} & & & & \cdots & & & & a_0 \\ \vdots & & & & & & & & & \\ 0 & \cdots & 0 & a_{-2n} & a_{-2n+1} & & \cdots & & & a_{-2n+jm} \\ \vdots & & & & & & & & & \\ 0 & \cdots & & & & 0 & a_{-2n} & \cdots & & a_{-2n+m} \\ & & & & & & & & & a_0 \end{bmatrix}}_{A} \bullet \underbrace{\begin{bmatrix} b_{-2n} \\ b_{-2n+1} \\ \\ \vdots \\ \\ b_0 \end{bmatrix}}_{h_{rx}} = \underbrace{\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{x}$$

or

$$A \bullet h_{rx} = x \tag{A5}$$

The receiver filter h_{rx} coefficients are determined by multiplying Equation A5 by the inverse matrix A^{-1}, expressed as

$$h_{rx} = A^{-1} \bullet x \tag{A6}$$

This result is applied to the GSM EDGE pulse shaping filter. If the receiver filter is defined every T/2 step, $h_{rx}(t)$ will be defined by a set of 11 coefficients ($b_{-10}, b_{-9}, \ldots, b_0$) (the receiver filter $h_{rx}(t)$ is noncausal here). Taking 11 samples spaced T/2 from the pulse shaping filter $h_{tx}(t)$ ($a_0, a_1, \ldots, a_{10}$) leads to

$$\begin{bmatrix} a_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{-2} & a_{-1} & a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{-4} & a_{-3} & a_{-2} & a_{-1} & a_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{-6} & a_{-5} & a_{-4} & a_{-3} & a_{-2} & a_{-1} & a_0 & 0 & 0 & 0 & 0 \\ a_{-8} & a_{-7} & a_{-6} & a_{-5} & a_{-4} & a_{-3} & a_{-2} & a_{-1} & a_0 & 0 & 0 \\ a_{-10} & a_{-9} & a_{-8} & a_{-7} & a_{-6} & a_{-5} & a_{-4} & a_{-3} & a_{-2} & a_{-1} & a_0 \\ 0 & 0 & a_{-10} & a_{-9} & a_{-8} & a_{-7} & a_{-6} & a_{-5} & a_{-4} & a_{-3} & a_{-2} \\ 0 & 0 & 0 & 0 & a_{-10} & a_{-9} & a_{-8} & a_{-7} & a_{-6} & a_{-5} & a_{-4} \\ 0 & 0 & 0 & 0 & 0 & 0 & a_{-10} & a_{-9} & a_{-8} & a_{-7} & a_{-6} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{-10} & a_{-9} & a_{-8} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_0 \end{bmatrix} \bullet \begin{bmatrix} b_{-10} \\ b_{-9} \\ b_{-8} \\ b_{-7} \\ b_{-6} \\ b_{-5} \\ b_{-4} \\ b_{-3} \\ b_{-2} \\ b_{-1} \\ b_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{A7}$$

The filter detennined in this manner has the impulse response shown previously. In this case, $a_0 = a_{-10} = 0$, hence, matrix A^{-1} is not defined. The inverse of a submatrix B of A where $B = A_{ij}$ with $(i,j) \in [1 \rightarrow 9]$ (that is, B is A with the first and last columns and rows removed) is required. Therefore, b_0, b_{-10} can be set to 0. The matrix A in Equation A7 becomes

$$A = \begin{bmatrix} -0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.031 & 0.001 & -0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.706 & 0.260 & 0.031 & 0.001 & -0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.706 & 0.927 & 0.706 & 0.260 & 0.0031 & 0.001 & -0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.032 & 0.261 & 0.706 & 0.927 & 0.706 & 0.260 & 0.031 & 0.001 & -0.000 & 0.000 & 0.000 \\ 0.000 & 0.001 & 0.032 & 0.261 & 0.706 & 0.927 & 0.706 & 0.260 & 0.031 & 0.001 & -0.000 \\ 0.000 & 0.000 & 0.000 & 0.001 & 0.032 & 0.261 & 0.706 & 0.927 & 0.706 & 0.260 & 0.031 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.001 & 0.032 & 0.261 & 0.706 & 0.927 & 0.706 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.001 & 0.032 & 0.261 & 0.706 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.001 & 0.032 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 1392 & 0 & 0 & -0 & 0 & -0 & 0 & -0 & 0 \\ -12288 & 34 & -0 & 0 & -0 & 0 & -0 & 0 & -34 \\ 33598 & -111 & 5 & -0 & 0 & -0 & 0 & -6 & 1507 \\ -44240 & 154 & -10 & 3 & -1 & 1 & -3 & 46 & -12473 \\ 33716 & -119 & 8 & -3 & 3 & -3 & 8 & -124 & 33796 \\ -12448 & 44 & -3 & 1 & -1 & 3 & -10 & 160 & -44348 \\ 1507 & -5 & 0 & -0 & 0 & -0 & 5 & -116 & 33682 \\ -36 & 0 & -0 & 0 & -0 & 0 & -0 & 36 & -12319 \\ 0 & -0 & 0 & -0 & 0 & -0 & 0 & -0 & 1395 \end{bmatrix}$$

$$h_{rx} = b = \begin{bmatrix} 0 \\ 0 \\ -0.0032 \\ 0.1382 \\ -1.1359 \\ 2.7318 \\ -1.1366 \\ 0.1387 \\ -0.0033 \\ 0 \\ 0 \end{bmatrix} \qquad h_{tx} = a = \begin{bmatrix} -0 \\ 0.0007 \\ 0.0315 \\ 0.2604 \\ 0.7057 \\ 0.9268 \\ 0.7057 \\ 0.2605 \\ 0.0315 \\ 0.0008 \\ 0 \end{bmatrix}$$

The recovered signal exactly matches the transmitted $3\pi/8$ offset 8-PSK constellation since the channel is ideal.