Simulation of Multipath Fading Effects in Mobile Radio Systems

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Abstract

RF signals transmitted via wireless mobile channels suffer from several effects like small-scale fading and signal dispersion and distortion. This paper reviews these effects and simulates Rayleigh and Rician multipath fading channels with a comparison between them in terms of the effect of RF signal random fluctuations, average received signal level, outage probability, and effect of Doppler shift. In addition to that, signal dispersion occurring to pulses transmitted through these types of channels has also been discussed and simulated.

1. Introduction

In mobile communication systems, the RF signal propagates from the transmitter to the receiver via multiple different paths due to the obstacles and reflectors existing in the wireless channel. These multipaths are caused by mechanisms of reflection, diffraction, and scattering from buildings, structures, and other obstacles existing in the propagation environment [1]. Multipath propagation is usually described by line of sight (LOS) path and non line of sight (NLOS) paths as shown in Figure 1.

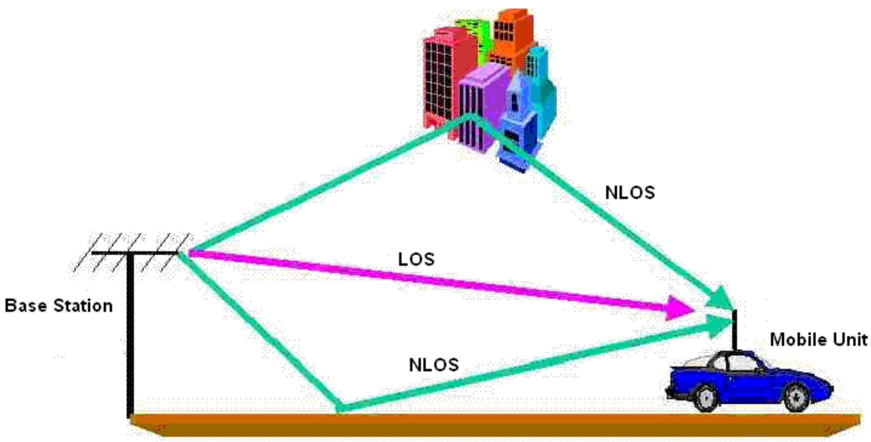


Figure 1: Multipath Propagation

When the mobile unit is considerably far from the base station, the LOS signal path does not exist and reception occurs mainly from the indirect signal paths. These multiple paths have different propagation lengths, and thus will cause amplitude and phase fluctuations and time delay in the received signal. Therefore, the main effect of multipath propagation can be described in terms of fading and delay spread [2].

When the waves of multipath signals are out of phase, reduction of the signal strength at the receiver can occur. This causes significant fluctuations in the received signal amplitude with time leading to a phenomenon known as *multipath fading* or *small scale fading*. A representation of multipath fading is shown in Figure 2.

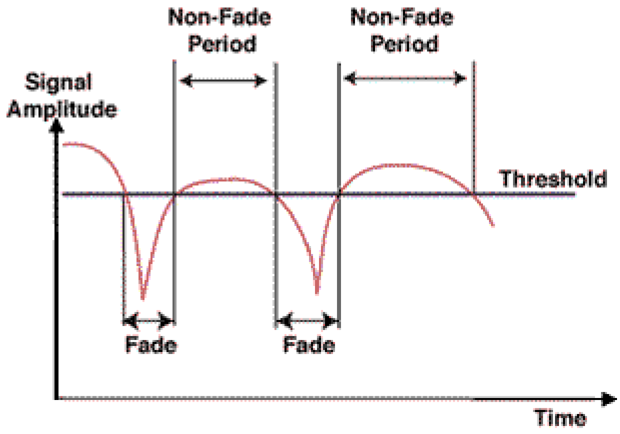


Figure 2: Representation of Multipath Fading

Small-scale fading is also called *Rayleigh fading* because if the multiple reflective paths are large in number and there is no line-of-sight signal component, the envelope of the received signal is statistically described by Rayleigh distribution. When there is a dominant nonfading signal component present, such as a line-of-sight propagation path, the small scale fading envelope is described by Rician distribution and, thus, is referred to as *Rician fading* [3].

When the mobile unit is moving, its velocity causes a shift in the frequency of the signal transmitted along each signal path. This phenomenon is known as the Doppler shift. Signals traveling along different paths can have different Doppler shifts, corresponding to different rates of change in phase. The difference in Doppler shifts between different signal components contributing to a single fading signal component is known as the *Doppler spread*. Channels with a large Doppler spread have signal components that are each changing independently in phase over time [4]. If the Doppler spread is significant relative to the bandwidth of the transmitted signal, the received signal will undergo *fast fading*. On the other hand, if the Doppler spread of the channel is much less than the bandwidth of the baseband signal, the signal undergoes *slow fading* [5]. So, the terms *slow* and *fast* fading refer to the rate at which the magnitude and phase change imposed by the channel on the signal changes.

Because multiple reflections of the transmitted signal may arrive at the receiver at different times, this can result in inter-symbol interference (ISI) due to the crashing of bits into one another. This time dispersion of the channel is called multipath *delay spread* and is an important parameter to asses the performance capabilities of wireless communication systems [6].

2. Modeling of Rayleigh Fading

As stated previously, Rayleigh fading results from the multiple NLOS paths of the signal propagating from transmitter to receiver. If the transmitted signal *s(t)* is assumed to be *unmodulated carrier*, then it may take the form:

$$s(t) = \cos(2\pi f_c t) \tag{1}$$

where f_c is carrier frequency of the radio signal.

After propagation over N reflected and scattered paths, the received signal may be considered as the sum of these N components with random amplitude and phase for each component. Thus, when the receiving station is stationary, the received signal *r(t)* can be written as [5]:

$$r(t) = \sum_{i=1}^{N} a_i \cos(2\pi f_c t + \varphi_i) \tag{2}$$

where a_i is a random variable corresponding to the amplitude of the i^{th} signal component, and φ_i is another uniformly distributed random variable corresponding to the phase angle of the i^{th} signal component.

Using the trigonometric identity:

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \tag{3}$$

equation (2) can be re-written in the form:

$$r(t) = \cos(2\pi f_c t).\sum_{i=1}^{N} a_i \cos\varphi_i - \sin(2\pi f_c t).\sum_{i=1}^{N} a_i \sin\varphi_i \tag{4}$$

Equation (4) can be expressed as [5]:

$$r(t) = X.\cos(2\pi f_c t) - Y.\sin(2\pi f_c t) \tag{5}$$

where:

$$X = \sum_{i=1}^{N} a_i \cos\varphi_i \tag{6}$$

$$Y = \sum_{i=1}^{N} a_i \sin\varphi_i \tag{7}$$

X and Y can be considered as two identical, independent Gaussian random variables when N tends to a large value [7].

Equation (5) represents the received RF signal when the receiver is stationary. If the mobile unit is moving with a speed of v meters per second relative to the base station, the received signal will acquire a Doppler shift in frequency. The maximum Doppler shift is given by [3]:

$$f_d = \frac{v}{c} f_c \tag{8}$$

The instantaneous Doppler shift in frequency is dependent on the angle of arrival of the incoming signal path component as shown in Figure 3 [3].

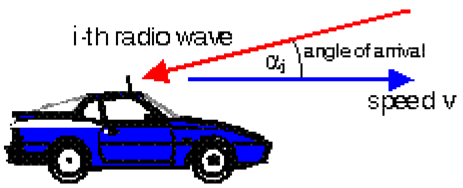


Figure 3: A Mobile Unit Moving at Speed *v*

The instantaneous value of the Doppler shift f_{di} can be expressed as [5]:

$$f_{di} = f_d \cos\alpha_i \tag{9}$$

where α_i is the angle of arrival for the i^{th} path signal component.

On the other hand, the instantaneous frequency of the received RF signal becomes [5]:

$$f_i = f_c + f_d \cos\alpha_i \tag{10}$$

Accordingly, the received signal can thereby expressed in the form:

$$r(t) = \sum_{i=1}^{N} a_i \cos(2\pi(f_c + f_{di})t + \varphi_i) \tag{11}$$

Equation (11) can alternatively be written in other form using the trigonometric identity (3):

$$r(t) = \cos(2\pi f_c t).\sum_{i=1}^{N} a_i \cos(2\pi f_{di} t + \varphi_i) - \sin(2\pi f_c t).\sum_{i=1}^{N} a_i \sin(2\pi f_{di} t + \varphi_i) \tag{12}$$

The received signal can also be formulated as [5]:

$$r(t) = \cos(2\pi f_c t).X(t) - \sin(2\pi f_c t).Y(t) \quad (13)$$

where:

$$X(t) = \sum_{i=1}^{N} a_i \cos(2\pi f_{di} t + \varphi_i) \quad (14)$$

$$Y(t) = \sum_{i=1}^{N} a_i \sin(2\pi f_{di} t + \varphi_i) \quad (15)$$

and $f_{di} = f_d \cos\alpha_i$.

X(t) and *Y(t)* are also known as the *in-phase* and *quadrature* components of the received signal respectively. It is seen from equation (13) that the received signal is like a quadrature modulated carrier. The envelope of the received signal is given by:

$$A(t) = \left[X(t)^2 + Y(t)^2\right]^{1/2} \quad (16)$$

It can be shown that the probability density function (pdf) of the envelope *A(t)* of the received signal is Rayleigh distributed [7].

The instantaneous power of the received signal is given by:

$$P(t) = X(t)^2 + Y(t)^2 \quad (17)$$

On the other hand, the average value of the received power P_{av} is the statistical mean of *P(t)*:

$$P_{av} = mean\left[P(t)\right] = \langle P(t)\rangle \quad (18)$$

At the receiver side, the in-phase and quadrature components *X(t)* and *Y(t)* can be obtained by demodulating the received signal r(t).

4. Modeling of Rician Fading

When the received signal is made up of multiple reflective paths plus a significant line of sight (non-faded) component, the received signal is said to be Rician faded signal because the pdf of the RF signal's envelope follows Rician distribution [3]. The received RF signal in this case can be written as:

$$r(t) = K_{LOS} \cdot \cos(2\pi(f_c + f_d)t) + \sum_{i=1}^{N} a_i \cos(2\pi(f_c + f_{di})t + \varphi_i) \quad (19)$$

where K_{LOS} is the amplitude of the direct (LOS) component, f_d is the Doppler shift in frequency along the LOS path, and f_{di} is the Doppler shift in frequency along the ith NLOS path signal component.

The received signal can be written in terms of in-phase and quadrature components as:

$$r(t) = K_{LOS} \cdot \cos(2\pi(f_c + f_d)t) + \cos(2\pi f_c t) \cdot X(t) + \sin(2\pi f_c t) \cdot Y(t) \quad (20)$$

where *X(t)* and *Y(t)* are given by equations (14) and (15) respectively.

5. Outage Probability

In a fading radio channel, the transmitted signal suffers deep fades that can lead to a complete loss of the signal or *outage* of the signal. The outage probability is a measure of the quality of the transmission in a mobile radio system. Outage occurs when the received signal power drops below a certain threshold level [5]. If *f(p)* is the probability density function of the received power , then the outage probability is calculated theoretically from [5]:

$$P_{outage} = \int_0^{P_{th}} f(p)dp \quad (21)$$

6. Signal Dispersion

In addition to fading, multipath radio channels affect also the shape of the signals sent from transmitter to receiver. If there are N indirect (NLOS) paths of the radio channel, and a pulse *s(t)* is being transmitted through this channel, then the received pulse will be:

$$r(t) = \sum_{i=1}^{N} a_i \cdot s(t - t_{di}) \quad (22)$$

where a_i is the amplitude of the i^{th} path component, and t_{di} is the delay time associated with the i^{th} path.

As shown from equation (22), the received pulse consists of N delayed versions of the transmitted pulse with different amplitudes. This will distort the received pulse by broadening and spreading it [5]. The dispersive characteristics of the multipath radio channel to a transmitted signal can be illustrated by evaluating the impulse response of the channel. When a very narrow pulse, like the delta function $\delta(t)$, is sent through the channel, then the received signal will approach the impulse response of the radio channel according to the convolution integral.

$$r(t) = h_c(t) \otimes s(t)$$

$$= \int_{-\infty}^{\infty} h_c(t - \tau) \cdot s(\tau)d\tau \quad (23)$$

where $h_c(t)$ is the impulse response of the channel.

When the transmitted pulse *s(t)* approaches the Dirac delta function $\delta(t)$, then the received signal *r(t)* will equal the impulse response of the channel $h_c(t)$.

$$r(t) = h_c(t)\Big|_{s(t)=\delta(t)} \tag{24}$$

7. Results of Simulation

To illustrate the fading effects of a typical mobile radio channel, some MATLAB script files were written and used for this purpose. The Rayleigh faded signal of equation (13) has been generated with a carrier frequency of 900MHz (typical GSM system), and a number of NLOS paths N=10. The amplitudes of the signal components a_i were generated as random numbers ranging from 0 to 1. The phase angles of the signal paths φ_i were generated as uniform distributed random numbers lying between 0 and 2π. The received signal vector was calculated with 2000 samples using a sampling time interval of 0.2 ns. The envelope of the received signal was generated by first extracting the in-phase component vector $\boldsymbol{X}$, and the quadrature component vector $\boldsymbol{Y}$ by demodulating the received signal vector $\boldsymbol{r}$ using the QAM method. The envelope vector $\boldsymbol{A}$ has been calculated using equation (16).

In a similar manner, the Rician faded signal was generated by means of equation (20) and using different values of the direct path signal component amplitude K_{LOS}.

Figure 4 presents the simulated Rayleigh and Rician faded signals and envelopes with a mobile speed v = 0 (stationary mobile unit). As shown from this simulation, the RF signals fluctuate randomly with time with different rates of change. The fluctuations in the Rayleigh faded envelope are higher when compared to the Rician faded envelope. In this case, it was assumed that the direct component gain of the Rician channel K_{LOS}=1. It is obvious that the lower rate of change in the Rician envelope is due to the presence of the direct path component in the received signal.

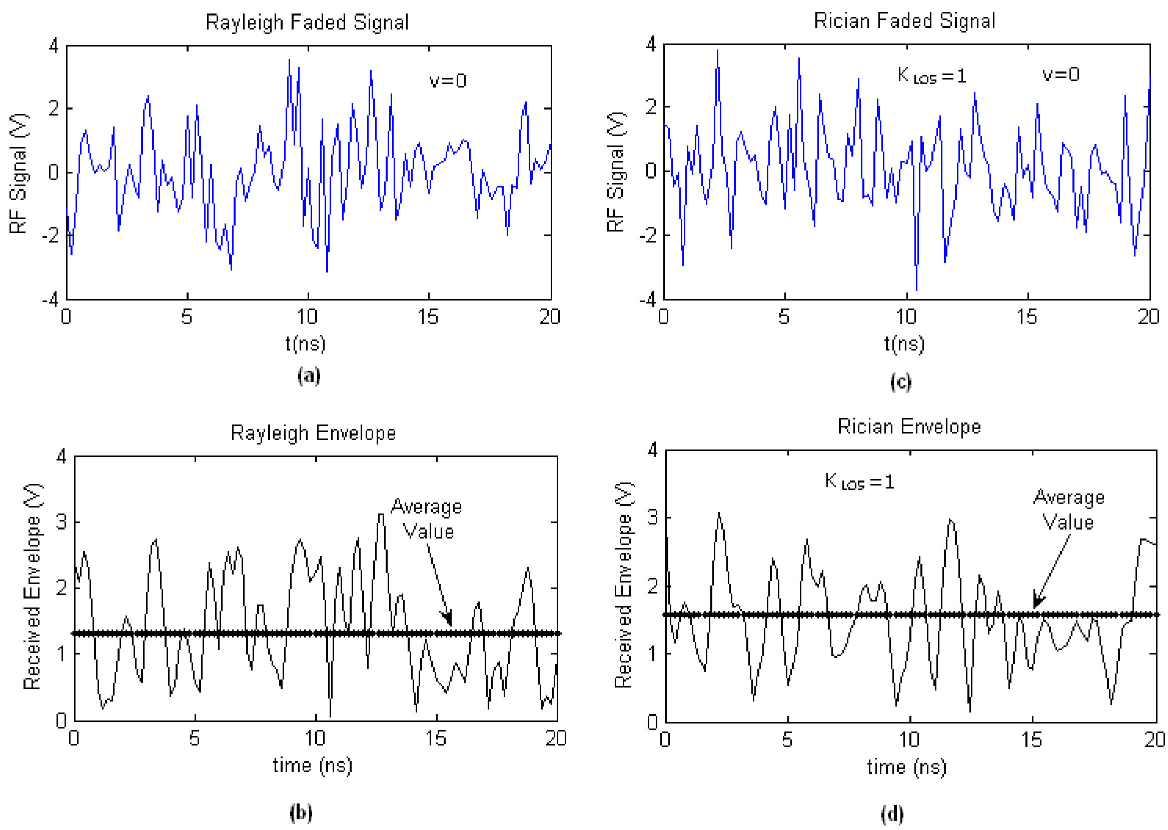


Figure 4: The Simulated Rayleigh and Rician Faded RF Signals and Their Envelopes

It can also be seen from Figure 4 that the average value of the Rician received signal envelope is relatively higher than that of the Rayleigh envelope. This surely depends on the value of the direct path gain K_{LOS}.

In Figure 5, the Rayleigh faded signal envelope was simulated with different mobile speeds. It can be noted how Doppler shift affects the received envelope by the increase in the rate of fluctuations when the mobile speed is increased.

Figure 6 depicts the effect of increasing the amplitude of the direct path component on the shape of the Rician faded received signal envelope. The higher the value of K_{LOS}, the higher the average value of the received signal power. Besides, the rate of random fluctuations in the received signal envelope is reduced by the increase of K_{LOS}.

In Figure 7, a comparison between the received power of the Rayleigh faded signal and the Rician faded signal is presented assuming stationary mobile receiver. The received power vector $\boldsymbol{P}$ was evaluated using equation (17), while the average power was calculated from equation (18). A similar threshold power level of -5 dBW was assumed in this simulation.

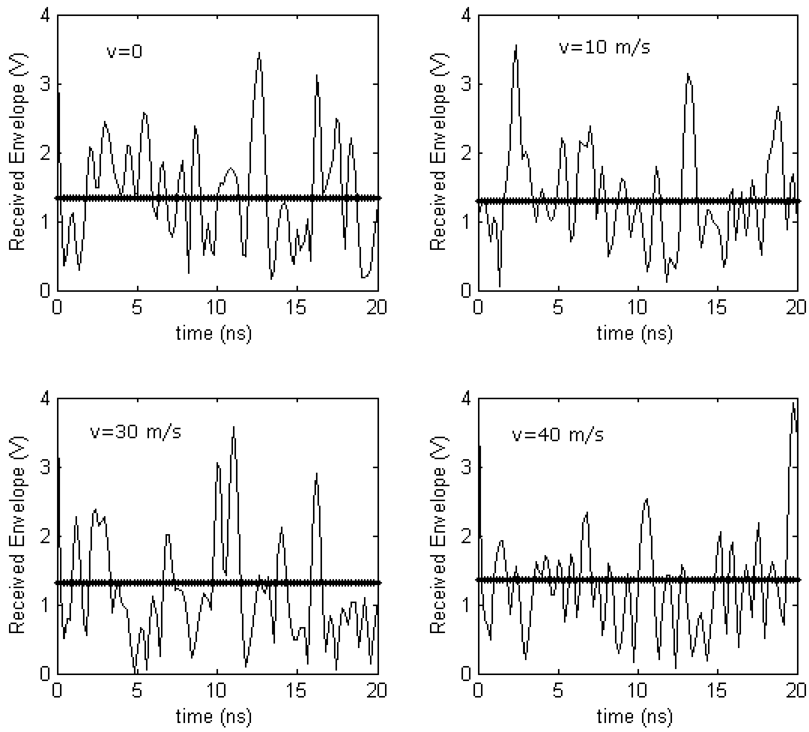


Figure 5: Simulated Rayleigh Faded Envelope with Different Mobile Speeds

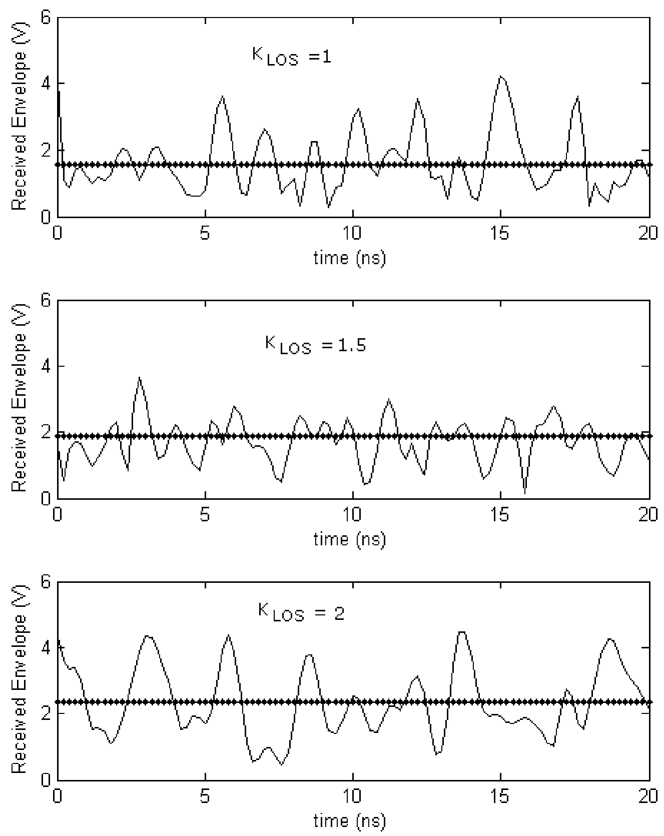


Figure 6: Simulation of the Rician Signal Envelope with Different Values of K_{LOS}

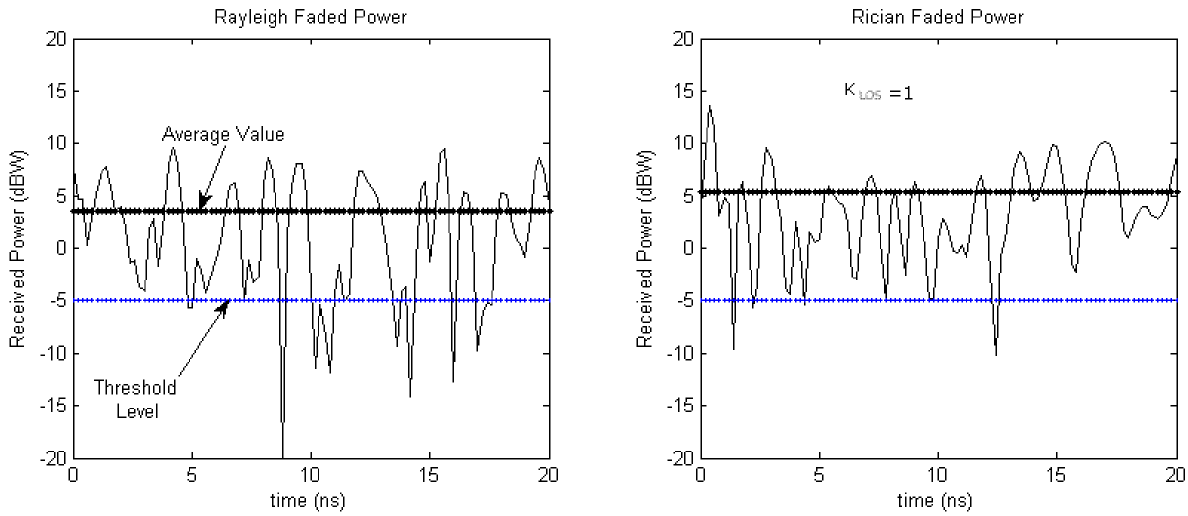
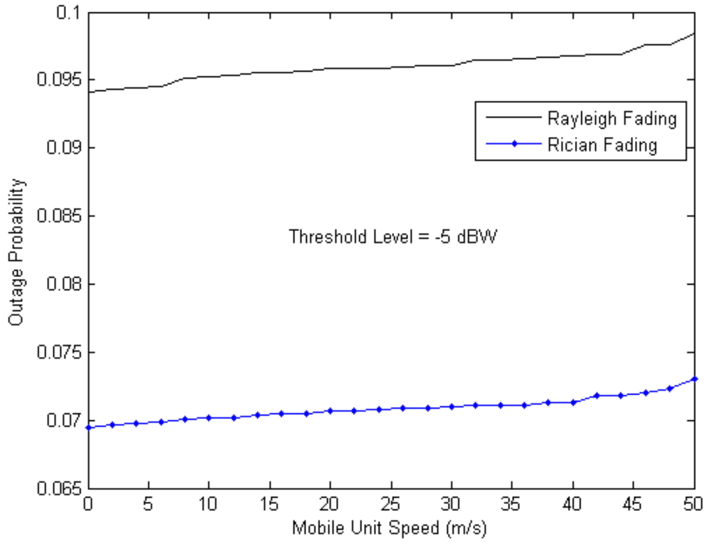


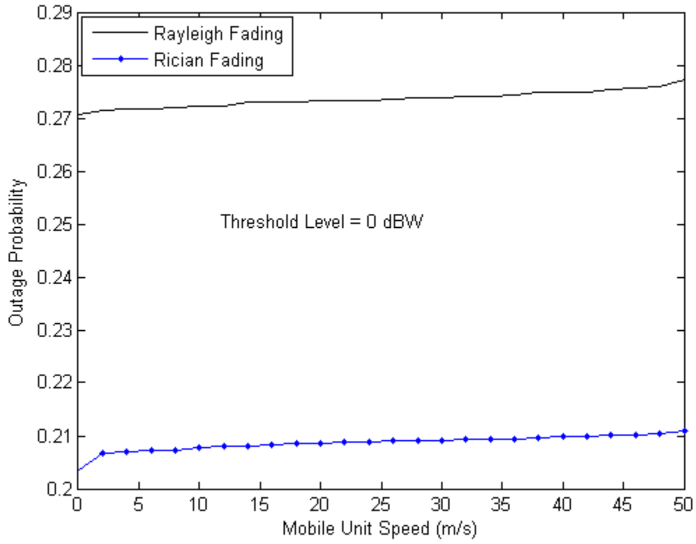
Figure 7: Simulated Received Power for the Rayleigh and Rician Fading Channels

It can be shown also from Figure 7 that the average received power is higher in the case of Rician fading channel, while the fade durations in the Rayleigh received signal are wider leading to larger outage probability.

The outage probability of the received signal power can be estimated by counting the sample values of the received signal power vector which are less than the threshold power level. The outage probability is then obtained by dividing the counted number of samples by the total number of samples of the simulated power vector [8]. Figure 8 shows the variation of the estimated outage probability with mobile speed for both the simulated Rayleigh and Rician faded signals. The outage probability is slightly increased with the increase of the mobile speed due to Doppler shift. The outage probability is generally higher in the case of Rayleigh fading.



(a)



(b)

Figure 8: Variation of Outage Probability with Mobile Speed for Two Different Threshold Power Levels

To demonstrate the effect of the fading channel on the shape of the transmitted pulses through it, a normalized Gaussian pulse has been sent through a multipath channel. The normalized Gaussian pulse takes the form [9]:

$$s(t) = e^{-\pi \cdot t^2 / \sigma^2} \tag{25}$$

where σ is a parameter representing the pulse width.

Figure 9 shows a typical Gaussian pulse with σ = 1ms sketched using MATLAB. This pulse has been transmitted through a multipath dispersive channel with N=10, and a maximum delay time of 2ms. The received signal was estimated using equation (22). Figure 10 presents four consequent simulations for the received pulse compared with the transmitted pulse in each case. It was assumed that the paths' amplitude coefficients of the channel take random values from 0 to 0.25, and paths' time delays take values from 0 to 2ms.

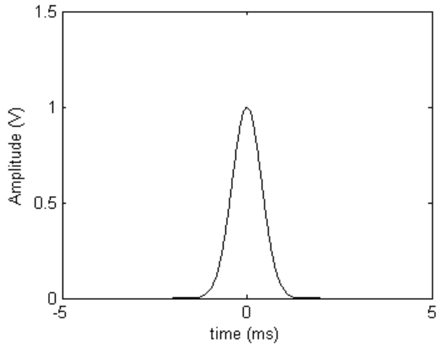


Figure 9: The Gaussian Pulse

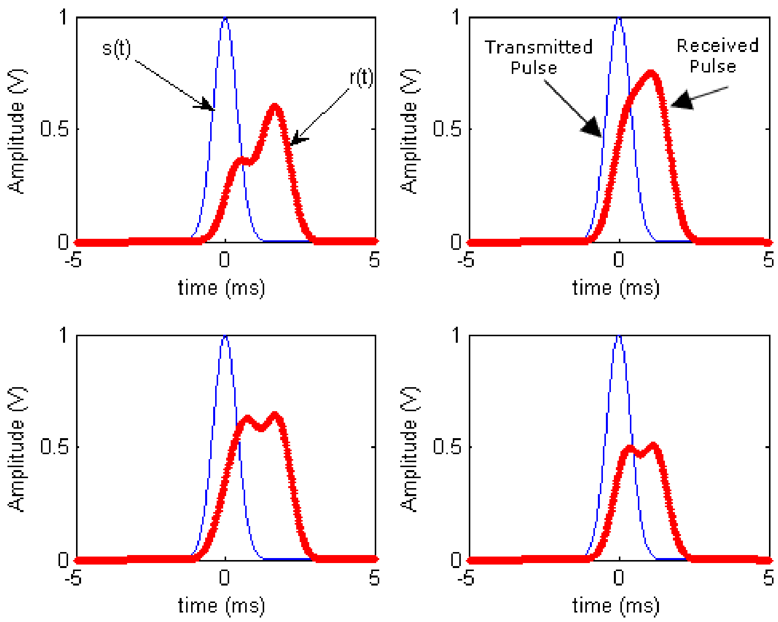


Figure 10: Comparison between Transmitted and Received Pulses via a Multipath Channel for Different Simulations

As shown from these simulations, the multipath channel has the effect of spreading or broadening the transmitted pulse, distorting it, and time shifting it. The random change in the shape of the received signal in these consequent simulations confirms the time varying characteristics of the mobile radio channel.

In order to compare the impulse responses of two types of mobile radio channels, a very narrow pulse (with σ = 10 μs) was sent through two mediums. The first medium represents a rural area, while the second one represents an urban region. In the rural area, it was assumed that the maximum delay time is 0.5ms, while the maximum delay time in the urban area was assumed to be 4ms. Figure 11 presents the received pulses in the two kinds of the channels which represent in this case the impulse responses of these channels as stated in equation (25). As shown from Figure 11, the received pulses reach in a shorter time in the case of the rural area. The urban area, on the other hand, causes spreading for the received pulses.

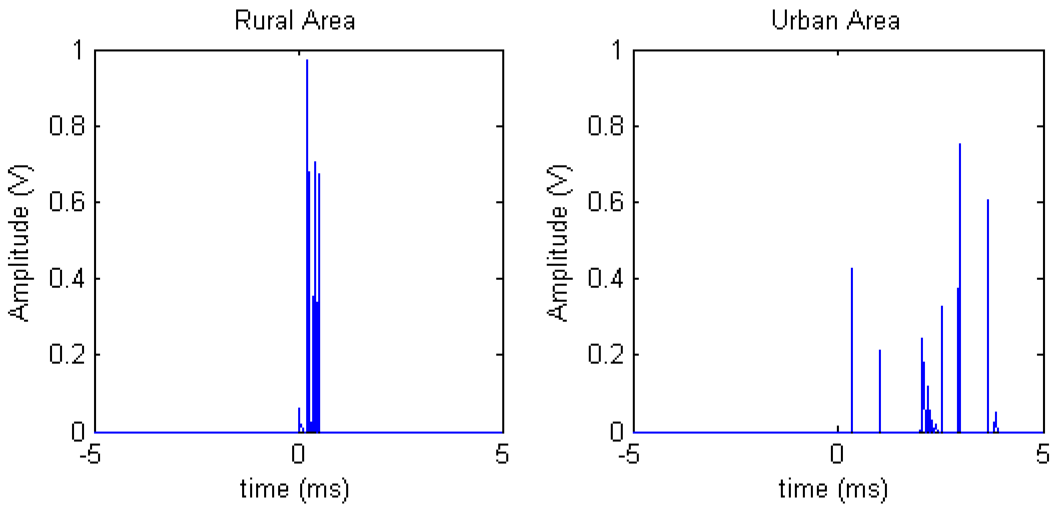


Figure 11: Comparison between the Channel Impulse Responses for the Rural and Urban Regions

8. Summary

This paper has reviewed the fading effects associated with mobile radio channels. MATLAB was used as a tool for simulating the received signal patterns for both Rayleigh and Rician fading channels. It was demonstrated that the degrading effects of the Rician channel on the received signals are relatively smaller than those in the Rayleigh channel. The examined parameters included average received power level, rate of received signal fluctuations, outage probability, and Doppler shift. The dispersive characteristics of multipath fading channels have also been simulated by showing the effects of these types of channels on the pulses transmitted throughout them.

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