TECHNICAL FEATURE

Phase and Amplitude Noise Due to Analog Control Components

This article presents the analysis necessary to calculate phase and amplitude noise due to the noise spectrum on a bias supply or control voltage in combination with component phase and amplitude sensitivity to this voltage. While analog phase shifter and attenuator noise contributions are caused by noise on the analog control line, microwave amplifiers contribute to the system noise in a similar fashion due to noise on the bias supplies. The phase and amplitude noise equations are used to determine the bias voltage spectral noise suppression requirements for typical microwave amplifiers vs. the control signal noise suppression required for typical analog phase shifters and attenuators. A conclusion is drawn that analog phase shifters and attenuators generally have much higher sensitivity to control voltage spectral noise and, thus, require significantly lower control voltage noise to achieve system noise contribution comparable to those of microwave amplifiers.

Analog control components such as voltage-controlled phase shifters or attenuators are sometimes employed in phased-array systems to add fine control or to simplify the circuitry from digital step control devices. Other systems employ these devices in analog phase nulling or automatic gain control loops. Noise on the analog control signal imparts phase and/or amplitude noise onto the processed signal. This noise is present only when the component is processing a signal, that is, it is not measurable without a signal at the input unlike the kT noise described by the noise figure or equivalent noise temperature. This article analyzes the magnitude of this effect.

PHASE AND AMPLITUDE NOISE

An unmodulated CW microwave signal containing amplitude and phase noise can be expressed as

$$V(t) = [V_o + \varepsilon(t)]\cos[2\pi f_o t + \Phi(t)] \qquad (1)$$

where

V_o = nominal peak voltage
f_o = nominal frequency
$\varepsilon(t)$ = amplitude time variation
$\Phi(t)$ = time variation of phase

$\varepsilon(t)$ and $\Phi(t)$ give rise to noise modulation sidebands about f_o. The time-averaged phase variation of $\Phi(t)$ at discrete modulation frequencies f_m represents the variation of $\Phi(t)$ in the frequency domain. The quantity $\Delta\Phi(f_m)$ is defined mathematically as the two-sided RMS phase variation in a 1 Hz bandwidth centered at offset frequency f_m with units of radians/$\sqrt{Hz}$. If both $\Sigma(\Delta\Phi)$ and $(\Delta\Phi)$ peak are << 1 radian over the range of f_m considered, then a small angle approximation can be applied to the Bessel function expansion of

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Equation 1 as shown in ***Appendix A***. This condition allows the interpretation that the power spectral density of the phase variation of $\Phi(t)$ defined as

$$S_\Phi(f_m) = \Delta\Phi^2(f_m)\ \text{rad}^2/\text{Hz} \tag{2}$$

is also equal to the ratio of two-sided phase noise to total signal power in a unity bandwidth at carrier offset frequency f_m.

The short-term (< 1 s) phase stability of a microwave signal is described by convention as the single-sided phase noise spectral density, which is the power in one sideband normalized to the sum of the power in the microwave carrier frequency f_o and both sidebands in a 1 Hz bandwidth centered at an offset frequency f_m from the microwave carrier frequency. This quantity ($\mathcal{L}(f_m)$) is expressed in decibels below the carrier power per hertz (dBc/Hz) and is defined as

$$\begin{aligned}\mathcal{L}(f_m) &= 10\log L(f_m)\\ &= 10\log\frac{\Delta\Phi^2(f_m)}{2}\\ &= 10\log\frac{S_\Phi(f_m)}{2}\ \text{dBc/Hz}\end{aligned} \tag{3}$$

since

$$\begin{aligned}S_\Phi(f_m) &= \Delta\Phi^2(f_m)\ \text{rad}^2/\text{Hz}\\ &= L(f_o+f_m) + L(f_o-f_m)\\ &\approx 2\,L(f_m)\end{aligned} \tag{4}$$

The quantity $\mathcal{L}(f_m)$ is convenient in system analysis because, as a bandwidth normalized power ratio, it is invariant to component gain or loss and changes in bandwidth as the analysis progresses through the system. The contributions from each component (in $L(f_m)$) need only to be summed to determine a system phase noise spectral density.

Similarly, the power spectral density of a small amplitude variation is defined as

$$S_\varepsilon(f_m) = \frac{\Delta\varepsilon^2(f_m)}{V_o^2}\ \text{Hz}^{-1} \tag{5}$$

where

$\Delta\varepsilon(f_m)$ = time-averaged two-sided RMS variation of $\varepsilon(t)$ at offset f_m from the microwave carrier

$\Delta\varepsilon^2(f_m)$ = power in both sidebands in a 1 Hz bandwidth

V_o^2 = total power in the microwave signal

The single-sided amplitude spectral noise density then is defined as

$$M(f_m) = 10\log\left[\frac{S_\varepsilon(f_m)}{2}\right]\text{dBc/Hz} \tag{6}$$

THE ANALOG PHASE SHIFTER

If K_Φ is the phase sensitivity of the microwave signal to control voltage in radians/volt, then

$$\frac{K_\Phi V_{noise}}{\sqrt{\text{Hz}}} = \Delta\Phi(f_m)\ \text{rad}/\sqrt{\text{Hz}} \tag{7}$$

is the microwave signal phase variation due to the spectral noise $V_{noise}/\sqrt{\text{Hz}}$ on the analog control voltage at carrier offset f_m and

$$\mathcal{L}(f_m) = 10\log\frac{1}{2}\left(K_\Phi V_{noise}\right)^2\ \text{dBc/Hz} \tag{8}$$

is the phase noise imparted to the microwave signal due to the phase shifter control voltage noise.

THE ANALOG ATTENUATOR

If K_ε is the microwave signal level sensitivity to control voltage in decibels per volt, then

$$\frac{K_\varepsilon V_{noise}}{\sqrt{\text{Hz}}} = 10\log\left(\Delta\text{gain}(f_m)\right)\text{dB}/\sqrt{\text{Hz}} \tag{9}$$

is the gain (or power level) variation of the microwave signal at carrier offset frequency f_m due to the spectral noise $V_{noise}/\sqrt{\text{Hz}}$ on the analog control voltage. Remembering that gain is defined as the squared ratio of output to input voltage,

$$\Delta\text{gain} = \frac{\left(\frac{V_{out}2}{V_{in}}\right)^2}{\left(\frac{V_{out}1}{V_{in}}\right)^2} = \frac{(V_{out}2)^2}{(V_{out}1)^2} \tag{10}$$

and

$$\begin{aligned}(\Delta\text{gain}-1) &= \frac{(V_{out}2)^2}{(V_{out}1)^2} - \frac{(V_{out}1)^2}{(V_{out}1)^2}\\ &= \frac{(V_{out}2 - V_{out}1)(V_{out}2 + V_{out}1)}{(V_{out}1)^2}\end{aligned} \tag{11}$$

or

$$\begin{aligned}(\Delta\text{gain}-1) &\approx \frac{(\Delta V_{out})(2V_{out})}{(V_{out})^2}\\ &\cong 2\frac{\Delta\varepsilon}{V_o}\end{aligned} \tag{12}$$

Thus,

$$S_\varepsilon(f_m) \cong \left(\frac{\left(\Delta\text{gain}(f_m)-1\right)}{2}\right)^2\ \text{Hz}^{-1} \tag{13}$$

and

$$M(f_m) = 10\log\frac{1}{2}\left(\frac{\left(\Delta\text{gain}(f_m)-1\right)}{2}\right)^2\ \text{dBc/Hz} \tag{14}$$

is the amplitude noise imparted to the microwave signal due to the attenuator control voltage noise.

THE MICROWAVE AMPLIFIER

A solid-state power amplifier imparts phase and amplitude noise as described previously onto a microwave signal as it is being amplified because minute changes in the gain or insertion phase modulate the signal as it passes through the amplifier. This multiplicative noise is present only at the amplifier output when a signal appears at the input. Additionally, thermal noise is generated in the amplifier and added to the signal. Additive noise appears at the output of a microwave amplifier even without a signal at the input. Intrinsic multiplicative noise has a spectral signature of f_m^{-1}, while additive noise is independent of Fourier frequency.

Generally, it is understood that the amplifier residual output noise is divided equally between amplitude and phase noise. However, amplitude noise can be converted to phase noise during the amplification process. This change occurs when the amplifier is operated in a nonlinear gain region (typically within 10 dB of the 1 dB gain compressed output) due to AM-to-PM conversion in the compressed amplifier stages. Many times, the amplifier multiplicative noise is domi-

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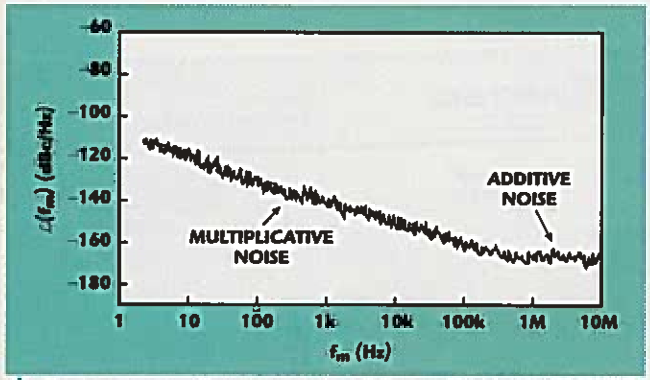
nated by power supply noise, not intrinsic device noise. The gate-to-source voltage for FETs or the base-to-emitter voltage for bipolar junction transistors (BJT) is the most sensitive bias voltage with regard to imparting phase and amplitude multiplicative noise onto a signal.

The multiplicative phase and amplitude noise of an amplifier arising from noise on the bias voltages as well as the 1/f noise due to carrier interactions in the FET depletion region or BJT junction are confined to within a few megahertz of the carrier. This noise has a fixed relationship to the carrier in decibels below the carrier power per hertz. Thus, the multiplicative noise reduces in absolute value as the output carrier power is reduced, dropping to zero with no RF drive. In contrast, the additive noise is constant in absolute value and its ratio to carrier power in decibels below the carrier power per hertz decreases as the output power increases. Of course, this relationship assumes that the noise mechanisms are unchanged as the RF drive level varies, which is not the case for power amplifiers. ***Figure 1*** shows a phase noise plot for a typical amplifier.



▲ ***Fig. 1** Intrinsic phase noise spectral density for a typical microwave solid-state amplifier.*

***Fig. 2** Multiplicative phase noise due to control voltage noise for typical analog attenuator and microwave amplifier pushing factors.* ▼

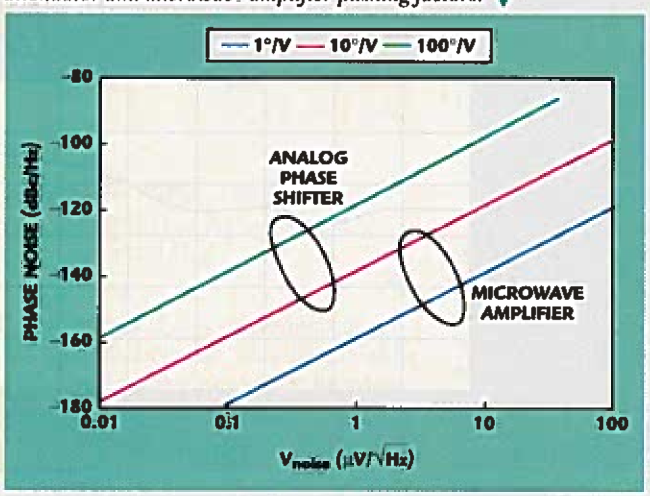
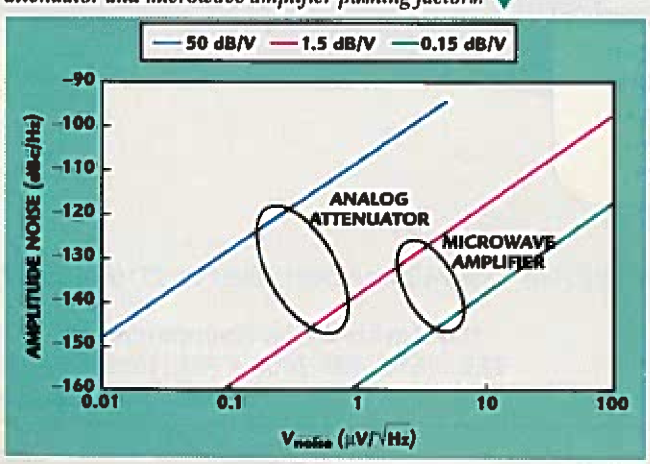


***Fig. 3** Multiplicative amplitude noise due to control voltage noise for typical analog attenuator and microwave amplifier pushing factors.* ▼



The residual phase and amplitude noise of an amplifier arising from noise on the bias voltages can be analyzed in the same fashion as described previously. The bias voltage may be considered the control voltage for either phase or amplitude modulation of the amplifier output signal. If the sensitivity of the amplifier output to each of the bias voltages is known, the $\mathcal{L}(f_m)$ and $M(f_m)$ in decibels below the carrier power per hertz may be expressed as a function of the spectral noise on each of the amplifier bias voltages using the expressions derived previously.

PRACTICAL IMPLICATIONS

It is instructive to apply this analysis to some commercial off-the-shelf components to quantify the magnitude of control voltage noise that might be tolerated in a microwave system. A survey of vendor catalogs shows that the response sensitivity can vary widely depending on device technology, circuit design techniques and operating frequencies. Designs that do not have a linear relationship between the control voltage and phase shift or attenuation response will introduce different amounts of noise depending on the steady state (or DC) value of the analog control voltage setting. Analog phase shifter sensitivity can vary from 4° to 100°/V while typical attenuator sensitivities can vary from 2 to 50 dB/V.

Packaged microwave amplifiers and transmit/receive (T/R) modules used in phased-array systems have internal bias regulators that reduce the noise sensitivity on the applied external bias voltage by one to four orders of magnitude. Typical sensitivities (or pushing factors) are 1° to 10°/V and 0.15 to 1.5 dB/V for both low noise and moderate (up to 15 W) power amplifiers having some internal regulation. These pushing factors demonstrate that control voltage noise on typical analog phase shifters and attenuators must be kept well below 1 $\mu V/\sqrt{Hz}$ RMS to realize phase and amplitude noise comparable to that produced in typical intrinsic microwave amplifiers.

The typical intrinsic phase and amplitude noise at the output of a 10 GHz solid-state amplifier (produced with noiseless (ideal) bias voltages) was shown previously. The intrinsic phase noise of a varactor phase shifter can be as low as –180 dBc/Hz and that of a PIN diode attenuator is at least –155 dBc/Hz. Thus, as shown in ***Figures 2*** and ***3***, the system noise clearly is limited by the noise on the bias and control voltages.

The amount of residual noise that can be tolerated by a system is usually less than –100 dBc/Hz for combined phase and amplitude noise at all offset frequencies f_m greater than some minimum value. The lowest f_m of inter-

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est varies according to the system application. A data communication system for a deep space probe may be sensitive to noise within 10 Hz of the carrier while a Doppler radar system may only be concerned with noise from a few kilohertz to several hundred kilohertz from the carrier. Ideally, the spectral noise density of a system is set by the master oscillator, which is the source of the transmitted signal and all LOs. Other components in the system then are expected to contribute minimally to the noise and, therefore, are required to be at last 6 to 10 dB lower than the master oscillator noise at all f_m of interest.

CONTROL VOLTAGE BANDWIDTH AND NOISE

It usually is impractical to filter the control voltage noise for phase shifters and attenuators because they are expected to respond to commands within a relatively short time period. Often the system phase and attenuation must be set and settled within a few microseconds. Likewise, a new control voltage value also must be set and settled quickly. The required frequency response of the control circuitry and the component control voltage sensitivity K_Φ or K_ε must be at least $1/(3 \times t_r)$ to support the required control voltage rise time. A 5 µs rise time t_r implies a control circuit bandwidth of at least 67 kHz to support this switching time. Since this frequency is higher than the minimum target Doppler of most radars or the frame rate of typical digital communication systems, it is imperative that the spectral noise floor of the control signal be limited without filtering (which would reduce the rise time).

It is possible to filter spectral noise on control signals while maintaining switching speed. The method requires active noise cancellation techniques or fast attack, adaptable bandwidth control loops. However, these techniques are difficult to implement if the control sensitivities vary significantly over the range of control voltage (such as is the case for unlinearized phase shifters and attenuators).

It is much easier to control the spectral noise on microwave amplifier bias voltages since these voltages usually are not switched (other than either on or off). For phased-array T/R modules, bias voltages may be filtered prior to a transistor switch, which is either saturated in the on state or biased at twice pinch off in the off state. The switch provides the required rise time to support the bias voltage transition from transmit to receive without contributing any additional noise. It is good practice not to switch the gate or base voltages of microwave amplifiers to various intermediate voltages to accommodate different modes of gain or power output as this procedure invites system phase and amplitude noise degradation arising from noise on the bias regulator output, which now is required to generate several discrete voltage levels and also have sufficient bandwidth to support the rise time requirement.

The frequency response of the analog control voltage sensitivity K_Φ or K_ε usually is not fully characterized by the component vendor. Thus, it is often necessary to measure K_Φ or K_ε over the range of f_m of interest to the particular system application. Care must be taken to assure that the small-signal AC value of K_Φ or K_ε is being measured at each DC setting. A 1 V swing in the AC control voltage probably is too coarse a measurement step, especially when the modulation of interest is caused by microvolt-level signals.

Usually, a digital-to-analog converter (DAC) will be employed to provide the multileveled control voltage necessary for analog phase shifters and attenuators employed in electronically steered phased arrays. The number of voltage levels (bits) and the update rate (rise time) are important design parameters. A 12-bit, 15 ns update rate DAC suitable for a rapidly steered array is available commercially with an output noise specification of 10 $nV/\sqrt{Hz}$. However, most DACs do not offer noise this low. Additionally, the size and cost of low noise DACs may make them unsuitable for phased-array applications.

Since the DAC employs an operational amplifier in its analog output circuitry, its output spectral noise can be described by classic operational amplifier theory. The standard expression for the total noise referred to the input of an operational amplifi-

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er at a given frequency f_m is

$$e_t = \sqrt{e_n^2 + R_n^2 i_n^2 + 4kTR_n}\ \text{V}/\sqrt{\text{Hz}} \quad (15)$$

where

e_n = voltage noise spectral density at f_m
i_n = current noise spectral density at f_m
R_n = sum of the effective inverting and noninverting input resistances
kT = Boltzman's constant × ambient temperature in kelvins

In general, the voltage noise dominates for R_n less than a few hundred ohms. At higher R_n, the kT thermal noise becomes the dominant term. For R_n greater than a few thousand ohms, the current noise dominates the operational amplifier noise. For low noise operational amplifiers, e_n can be as low as 1 nV/$\sqrt{\text{Hz}}$ and i_n as low as 1.2 pA/$\sqrt{\text{Hz}}$ at f_m = 1 kHz. These values increase in magnitude below some corner frequency as $1/f_m$. The corner frequency for silicon, FET input, operational amplifiers is approximately 100 Hz for voltage noise and approximately 200 Hz for current noise. Thus, for f_m = 1 kHz, the device is said to be operating at the noise floor since f_m is above the corner frequencies.

When referred to the output, the noise described previously increases by the voltage gain of the operational amplifier. However, practical circuit limitations having to do with analog and digital grounds, electromagnetic interference shielding and DC power supply noise often cause the noise on a DC-coupled control signal from an operational amplifier or DAC to increase significantly in practice, perhaps by an order of magnitude or more. Thus, a control voltage noise floor less than 100 nV/$\sqrt{\text{Hz}}$ is difficult to achieve in the field.

CONCLUSION

This article has presented the analysis necessary to calculate system phase and amplitude noise contribution due to the noise spectrum on a bias supply or control voltage of a microwave amplifier or analog control component in combination with component phase and amplitude sensitivity to this voltage. The phase and amplitude noise equations are used to determine the bias voltage spectral noise suppression requirements for typical microwave amplifiers vs. the control signal noise suppression required for typical analog phase shifters and attenuators. Analog phase shifters and attenuators typically require control voltage spectral noise to be much less than 1 μV/$\sqrt{\text{Hz}}$ to limit degradation of system phase and amplitude noise. Rapidly switched systems may need to limit use of analog phase shifters and attenuators and avoid switching the gate or base voltage of transistors to achieve low phase and amplitude noise. ■

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APPENDIX A

A microwave signal containing phase modulation may be expressed as

$$V(t) = V_o \cos[2\pi f_o t + \Phi(t)] \quad (A1)$$

For sinusoidal phase modulation, $\Phi(t)$ may be written as

$$\Phi(t) = \beta \sin 2\pi f_m t \quad (A2)$$

where

$\beta = \Delta\Phi$ peak at modulating frequency f_m (A3)

Substituting into Equation A1 and employing a Bessel function expansion yields[1]

$$\begin{aligned} V(t) = V_o[&J_0(\beta)\cos 2\pi f_o t \\ &- J_1(\beta)\cos 2\pi(f_o - f_m)t \\ &+ J_1(\beta)\cos 2\pi(f_o + f_m)t \\ &+ J_2(\beta)\cos 2\pi(f_o - 2f_m)t \\ &+ J_2(\beta)\cos 2\pi(f_o + 2f_m)t \\ &- J_3(\beta)\cos 2\pi(f_o - 3f_m)t \\ &+ J_3(\beta)\cos 2\pi(f_o + 3f_m)t + \ldots] \end{aligned} \quad (A4)$$

where

$$J_0(\beta) \cong 1 - \left(\frac{\beta}{2}\right)^2$$

and

$$J_n(\beta) \cong \left(\frac{1}{n!}\right)\left(\frac{\beta}{2}\right)^n \text{ for } n \neq 0 \quad (A5)$$

For small-angle modulation, $\beta << 1$ and

$$J_0 \cong 1,\ J_1 \cong \frac{\beta}{2} \text{ and } J_n \cong 0 \text{ for } n \geq 2 \quad (A6)$$

Thus, for small-angle modulation, only two sidebands contain any significant power and the microwave signal can be expressed as

$$V(t) = V_o\left[\cos 2\pi f_o t - \left(\frac{\beta}{2}\right)\cos 2\pi(f_o - f_m)t + \left(\frac{\beta}{2}\right)\cos 2\pi(f_o + f_m)t\right] \quad (A7)$$

The power in one sideband is proportional to

$$\left(V_o\frac{\beta}{2}\right)^2$$

and the power in the carrier and all sidebands is proportional to $(V_o)^2$. Thus, the ratio of two-sided phase modulation to total power at frequency f_m is

$$\begin{aligned} S_\Phi(f_m) &= \frac{2\left(V_o\frac{\beta}{2}\right)^2}{(V_o)^2} \\ &= \frac{\beta^2}{2} \\ &= \frac{\Delta\Phi^2(\text{peak})}{2} \\ &= \Delta\Phi^2(\text{RMS}) \end{aligned} \quad (A8)$$

REFERENCE

1. Taub and Schilling, *Principles of Communication Systems*, Chapter 4, McGraw-Hill Inc., New York, NY, 1986.