Analysis and Design of the Rectangular Microstrip Patch Antennas for TM_{0n0} operating mode

Pasquale Dottorato RF & Microwave Designer Engineer
email: pasquale.dottorato@alice.it http://it.linkedin.com/in/microwavengineer
Ph.: +39(0)524202229 mobile ph.:+393318451342

Abstract

The analysis of rectangular microstrip patch antenna (RMPA) operating on the TM_{0n0} modes is presented. The characteristic parameters such as dielectric substrate, ground plane aspect ratio, input impedance as a function of the feed point position and the frequency effects have been theoretically investigated for these modes. The Finite Element Method and MATLAB model have been used for optimize and design RMPA operating at TM_{010} mode. The antenna has been carry out on the FR4 substrate and the characteristics has been measured with Vector Network Analyzer.

1. Introduction

The microstrip patch antenna is a popular printed resonant antenna for narrow-band microwave wireless links that require semi-hemispherical coverage. Due to its planar configuration and ease of integration with microstrip technology, the microstrip patch antenna has been heavily studied and is often used as elements for an array. However, they suffer from the disadvantage of low operating bandwidth [1]-[3]. Cavity model has been somewhat successful for calculating input impedance and radiation pattern [4]. Although proven to be useful for predicting input impedance, this method does not rigorously account for the presence of surface waves on the antenna substrate. Furthermore, it does not take into account existence of other modes with a dominant mode which deforms the surface currents and hence deforms the pattern lobs, and does not include feed effects. This paper presents the use of cavity model to analyze the rectangular microstrip antenna RMPA because this method can give a good pre-estimation on rectangular patch performance [5]. RMPA is characterized by its length L, width W, and thickness h, as shown in Fig. 1. The microstrip antenna treated as a cavity bounded by a magnetic wall along the edge, and by electric wall from above and below
suggest and the ***E*** has only the z component and ***H*** only the xy-components [4,6].

2. Theoretical formulation

Resonance frequency

Fig. 1 Rectangular Microstrip Patch Antenna geometry

The resonance frequency f_{nmp} depends on the patch size, cavity dimension and the filling dielectric constant [7,8] as follows:

$$f_{nmp} = \frac{k_{nmp} c}{2\pi\sqrt{\varepsilon_r}} \qquad (1)$$

where m,n,p = {0,1,2..} are, respectively, number of half-cycle field variation along the x,y,z directions; k_{nmp} is wave number @ mode n,m,p; ε_r is relative dielectric constant and :

$$k_{nmp} = \sqrt{\left(\frac{m\pi}{W}\right)^2 + \left(\frac{n\pi}{L}\right)^2 + \left(\frac{p\pi}{h}\right)^2} \quad (2)$$

with:

$$k_x = \left(\frac{m\pi}{W}\right); k_y\left(\frac{n\pi}{L}\right); k_z\left(\frac{p\pi}{h}\right)$$

W is radiating edge; L is non radiating edge; h is thickness substrate. The ratio W/L=1,5 is usually chosen, and L/W << h; it gives good performance according to the side lobe appearances [8]. In practice the fringing effect causes the effective distance between the radiating edge of the patch to be slightly greater than L. therefore, the actual value of the resonant frequency is slightly less than f_r. For TM_{0n0} mode, the length of non radiating rectangular patch's edge at a certain resonance frequency and dielectric constant according to eq, (1),(2) becomes:

$$L_n = \frac{nc}{2 f_r \sqrt{\varepsilon_r}} \quad (5)$$

taking into account the effect of fringing field, the effective dielectric constant for TM_{0n0} mode is derived using [9,10]:

$$\varepsilon_{ne} = \left(\frac{\varepsilon_r + 1}{2}\right) + \left(\frac{\varepsilon_r - 1}{2}\right)\left[1 + 10\left(\frac{h}{W_n}\right)\right]^{-0,5} \quad (6)$$

if (W/h)>1 then:

$$\Delta L_n = 0,412h \frac{(\varepsilon_{ne} + 0,3)\left(\frac{W_n}{h} + 0,264\right)}{(\varepsilon_{ne} - 0,258)\left(\frac{W_n}{h} + 0,813\right)} \quad (7)$$

$$L_{ne} = L_n + 2\Delta L_n \quad (8)$$

$$f_{nre} = \frac{nc}{2L_{ne}\sqrt{\varepsilon_{ne}}} \quad (9)$$

where ΔL is line extension; ε_{en} is effective dielectric constant. It can be demonstrated that it is possible to minimize the difference between the two frequencies, f_{re} and f_r, by modulating the length s, where:

$$s = 2\left(\frac{L_{eff} - L}{L}\right)^{1,5}$$

2.1 The Field Model

The electric field is assumed to act entirely in the z-direction and to be a function only of the x and y coordinate; i.e.

$$\vec{E} = \hat{z} E_z(x, y) \quad (10)$$

The field configurations within the cavity can be found using the vector potential approach [13]. The dielectric material of the substrate is assumed to be truncated and not extended beyond the edge of the patch. The potential vector A_z must satisfy the homogenous wave eq.:

$$\nabla^2 A_z + k^2 A_z = 0 \qquad (11)$$

subject to the boundary conditions of

$$\begin{cases} H_z([0,h],0,[0,W]) = H_z([0,h],L,[0,W]) = 0 \\ H_y([0,h],[0,L],0) = H_y([0.h],[0,L],W) = 0 \\ E_y(0,[0,L],[0,W]) = E_y(h,[0,L],[0,W]) = 0 \end{cases}$$

The final form for the vector potential A_z within the cavity is:

$$A_z = A_{mnp}\cos(k_x x)\cos(k_y y)\cos(k_z z) \qquad (12)$$

where A_{nmp} represent the amplitude coefficient of each m,n,p modes. The outward current flowing on the perimeter of the patch must be zero (since the patch boundary is an open-circuit). So:

$$H_z = \frac{\partial E_z}{\partial n} = 0$$

with **n** is the outward normal vector at the perimeter of the patch. By using the separation of variables, the electric field of the m, n and p mode number is [13]:

$$E_z = -j\frac{1}{\omega\varepsilon\mu}\left(\frac{\partial^2}{\partial^2 z} + k^2\right)A_z$$

$$E_z = -j\frac{(k^2 - k_z^2)}{\omega\varepsilon\mu}A_{nmp}\cos(k_x x)\cos(k_y y)\cos(k_z z)$$

To determine the dominant mode with the lowest resonance, we need to examine the resonant frequencies. The mode with the lowest order resonant frequency is referred to as the dominant mode. Therefore by TM_{0n0} mode is:

$$E_{zn} = E_{on}\cos\left(\frac{n\pi}{L}y\right) \qquad (14)$$

Similarly for H field:

$$E_{zn} = E_{on}\sin\left(\frac{n\pi}{L}y\right) \qquad (15)$$

where E_{on}=-jωεrA_{0n0} and H_{on}=(nπ/L)A_{0n0}. The electric field structure within the substrate and between the radiating element and the ground plane is undergoes a phase reversal along the length but it is uniform along its width. The phase reversal along the length is necessary for the antenna to have broadside radiation characteristics.

2.2 Far Field evaluation

In order to calculate the far field, the aperture model is used; edge effects can be taken into account using diffraction theory [14,15]. The resonator surface is considered to be a set of four slots of width 2a [11,14]. By using Green's function the following general form of the far field for any m,n,p modes.

$$\vec{E}(\vec{r}) = \frac{jke^{-jkr}}{2\pi r}\left[E_\vartheta \hat{i}_\vartheta + E_\varphi \hat{i}_\varphi\right] \qquad (16)$$

The total field is the sum of the two element array with each element representing one of the slot. Since the slot are identical this is accomplished by using an array factor for two slot.

- Radiating Slots -

The far field radiated by each slot, using the equivalent current densities [15]:

$$\begin{cases} E_\varphi = j\dfrac{hWE_o e^{-jkr}}{2\pi r}\left[\sin(\vartheta)\sin c(X)\sin c(Z)\right] \\ E_\vartheta = j\dfrac{hWE_o e^{-jkr}}{2\pi r}\left[\cos(\vartheta)\cos(\varphi)\sin c(X)\sin c(Z)\right] \\ E_r = H_r = 0; H_\vartheta = -\dfrac{E_\varphi}{\varsigma}; H_\varphi = \dfrac{E_\vartheta}{\varsigma} \end{cases}$$

where:

$$\begin{cases} X = \dfrac{k_o h}{2}\sin(\vartheta)\cos(\varphi) + \dfrac{m\pi}{2} \\ Z = \dfrac{k_o W}{2}\cos(\vartheta) - \dfrac{n\pi}{2} \end{cases} \qquad (18)$$

The array factor for the two elements, of the same magnitude and phase, separated by a distance L_{eff} along the y direction is:

$$(AF)_y = 2\cos\left(\frac{k_o L_{eff}}{2}\sin(\vartheta)\sin(\varphi)\right) \qquad (19)$$

Fig. 2 Radiating slots and θ,φ way specification by RMPA

- Non Radiating Slots -

The field radiated by the non radiating slots, each of effective length L_{eff} and high h, is [15]:

$$E_\vartheta = -\frac{khL_{eff}E_o e^{-jkr}}{2\pi r}\left[Y\cos(\varphi)\sin c(X)\frac{\cos(Y)}{\left(Y^2 - \left(\frac{\pi^2}{4}\right)\right)}\right]e^{j(X+Y)}$$

$$E_\varphi = \frac{khL_{eff}E_o e^{-jkr}}{2\pi r}\left[Y\sin(\varphi)\cos(\vartheta)\sin c(X)\frac{\cos(Y)}{\left(Y^2 - \left(\frac{\pi^2}{4}\right)\right)}\right]e^{j(X+Y)}$$

with

$$Y = \frac{kL_{eff}}{2}\sin(\vartheta)\sin(\varphi) \qquad (20)$$

Since the two non radiating slots form an array of two elements, of the same magnitude but of apposite phase, separated along the x-axis by a distance W, the array factor is:

$$(AF)_x = 2j\sin\left(\frac{kW}{2}\cos(\vartheta)\right) \qquad (21)$$

2.3 Input Impedance and Feed Point position

Harrington [6] gives the radiation conductance for a parallel-plate radiator as:

$$G = \frac{\pi W}{\varsigma\lambda}\left[1 - \frac{(kh)^2}{24}\right] \qquad (22)$$

when λ is the free space wavelength. He capacitive susceptance relates to the effective strip extension [16]:

$$B = 0{,}01668\frac{\Delta LW}{h\lambda}\varepsilon_{eff} \qquad (23)$$

But for general case the element Zij of the Z-matrix, is the impedance between the ith and jth pert of the patch, and is given by:

$$Z_{ij} = \frac{1}{W_i W_j}\iint_{w_i w_j} G(x_i y_i / x_j y_j)\, ds_i ds_j \qquad (24)$$

Where (x_i,y_i) and (x_j,y_j) denote the location of the two ports widths W_i and W_j, respectively; the Green function is given by:

$$G(x,y;x_o,y_o)=$$
$$\frac{j\omega\mu h}{LW}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\sigma_m\sigma_n\cos(k_x x)\cos(k_y y)\cos(k_x x_o)\cos(k_y y_o)}{k_x^2+k_y^2-k^2}$$

In particular the impedance varies from zero in the center to the edge resistance approximately as:

$$R_i = R_e\cos^2\left(\frac{\pi y}{L}\right) \qquad 0\le y\le L/2 \qquad (25)$$

when R_i is the input resistance, R_e the input resistance at the edge, and y the distance from the patch center. The feed location does not significantly affect the resonant frequency. Hence:

$$y=\frac{L}{\pi}\cos^{-1}\left(\sqrt{\frac{R_i}{R_e}}\right) \qquad (26)$$

This feed location is refers to a linearly polarized antenna, while, the R_e resistance is given:

$$R_e=\frac{1}{2(G_r+G_m)} \quad \text{and} \quad G_m=G_rF_g \qquad (27)$$

where

G_r is calculated with (22) and:

$$F_g=J_o(l)+\frac{p^2}{24-p^2}J_2(l)$$

The $J_n(.)$ are first type n-order Bessel function's where:

$$l=k(L+\Delta L)$$
$$p=k\Delta L$$

2.4 Fringing Field Extension Field

The extension ΔL in the length of the patch dimension to account for the fringing field is given [8], [9] by:

$$\Delta L=hg_1g_2g_5/g_4 \qquad (28)$$

with

$$g_i=\Im_i(\varepsilon_e,\varepsilon_r,g_2,W,h)\ for\ i=\{1,..,5\} \qquad (29)$$

2.5 Effective Width

The effective width We is determined [16], [17] by using the following expression:

$$W_e=2\pi h/[\ln(hF/W'+\sqrt{1+(2h/W')^2}] \qquad (30)$$

Where

$$F=6+(2\pi-6)\exp[(-\frac{4\pi^2}{3})(\frac{h}{W'})^{3/4}$$

$$W'=W+(\frac{t}{\pi})\left\{1+\ln\left[\frac{4}{\sqrt{\left(\frac{t}{\pi}\right)^2+\frac{(1/n)^2}{[(\frac{W}{t})+1.1]^2}}}\right]\right\}$$

Here, W is width of the patch , h is the thickness of the substrate, and t is the thickness of the metallic patch conductor.

2.6 The Frequency dependence

The frequency dependence of the width and permittivity is taken into account by modifying expression for ε_e and W_e [8], [12] and [17]:

$$\varepsilon_e(f)=\varepsilon_r-(\varepsilon_r-\varepsilon_e)/(1+P) \qquad (31)$$

where P is functionally of n-th frequency and u:

$$P = \wp(f_n, u) \qquad (32)$$

and

$$f_n = fh(\text{in GHz - mm})$$
$$u = [W + \frac{W' - W}{\varepsilon_r}]/h \qquad (33)$$

While for effective width, the expression is given by:

$$W_e(f) = \frac{W}{3} + (R_W + P_W)^{1/3} - (R_W - P_W)^{1/3} \qquad (34)$$

with

$$\begin{cases} S_W = s(f, \varepsilon_e) \\ Q_W = q(S_W, W) \\ P_W = p(S_W, W, W_e) \\ R_W = r(P_W, Q_W) \end{cases} \qquad (35)$$

2.7 Attenuation Constant Effect

The attenuation constant in RMPA is given:

$$\alpha = \alpha_d = \alpha_{cs} + \alpha_{cg}$$

Where to account the dielectric, conductor strip and ground plane losses. In according with [17] the attenuation is function of dielectric constant, frequency , size, surface metal type and conductivity.

2.8 Power Radiation Pattern

The radiated power from the patch is obtained by using the concept of the magnetic wall along the periphery, you see §§ 2.1-2.2. At the magnetic wall, the induced magnetic current density is:

$$\vec{J} = \hat{n} \times \vec{E}$$

where **n** is the unit normal to the magnetic wall. The total power dissipated by the antenna is $P_T=P_r+P_c+P_d$; the radiated power out side the antenna surface is given by[11]:

$$P_r = \frac{1}{\varsigma} \int_0^{2\pi} \int_0^{\pi/2} |E_\vartheta|^2 + |E_\varphi|^2 r^2 \sin(\vartheta) d\vartheta d\varphi \qquad (36)$$

while, the power losses inside the dielectric is given by [4]:

$$P_d = \frac{\omega \varepsilon_o \varepsilon_r tg(\delta_e)}{2} \iiint_V \vec{E} \bullet E^* dV \qquad (37)$$

The dielectric loss in the dielectric material and the conductor losses are taken into account by modifying the loss tangent of the substrate to the effective loss tangent; consequently, $\tan(\delta_e)$ is greater than $\tan(\delta)$ as it accounts for both dielectric and conductor losses:

$$\tan(\delta_e) = \frac{P_d + P_c}{P_d} \tan(\delta) \qquad (38)$$

By equations (36) and (18):

$$P_r = \frac{|V_o|}{2\varsigma\pi} \int_0^{2\pi} \left[\frac{\sin(\frac{k_o W}{2} \cos(\vartheta))}{\cos(\vartheta)} \right]^2 \sin^3(\vartheta) d\vartheta \qquad (39)$$

The directivity is given by:

$$D = \frac{4\pi}{\iint P_n(\theta, \varphi) d\Omega} \qquad (40)$$

$P_n(\theta,\varphi)$ gives the normalized power in the (θ,Φ) direction. The efficiency of RMPA is given by:

$$\eta = \frac{P_r}{P_r + P_d + P_c} \qquad (41)$$

The power losses inside the conductor surface of radiator and the ground plane is given by [4]:

$$Pc = 2\frac{R_S}{2}\iint\left(H_x^2 + H_y^2\right)dxdy \qquad (41.1)$$

with:

$$R_S = \sqrt{\frac{\varpi\mu_o\mu_r}{2\sigma}}$$

and R_s is skin-effect surface resistance of the conductor.

while total quality factor Q_T is:

$$Q_T = \frac{\omega W_T}{P_T} \qquad (42)$$

where, W_T is total stored energy written as:

$$W_T = \frac{\varepsilon_o\varepsilon_r}{2}\iiint_V |E_z|^2 dV \qquad (42.1)$$

and P_T is total power. The 2:1 VSWR of the rectangular patch is related to the quality factor Q that include the space-and surface-wave radiations[16]:

$$BW = \frac{1}{\sqrt{2}Q} = \frac{16C_1 phW}{3\sqrt{2}\eta\varepsilon_r L\lambda} = \frac{VSWR-1}{Q\sqrt{VSWR}} \qquad (43)$$

where

$$p = 1 - \frac{0,16605(kW)^2}{20} + \frac{0,02283(kW)^4}{560} - \frac{0,09142(kL)^2}{10} \qquad \eta = \frac{4C_1}{4C_1 + 3\pi kh\mu_r\left(1-\left(\frac{1}{n^2}\right)\right)^3} \qquad (44)$$

$$C_1 = 1 - \frac{1}{n^2} + \frac{0,4}{n^4}; n = \sqrt{\varepsilon_r\mu_r}$$

2. 9 Ground plane dimensions

The cavity model is well applicable to infinite ground planes. However, for practical considerations, it is essential to have a finite ground plane. It has been shown by [15,16] that similar results for finite and infinite ground plane can be obtained if the size of the ground plane is greater than the patch dimensions by approximately six times the substrate thickness all around the periphery. Hence, the ground plane dimensions would be given as:

$$\begin{cases} L_g = 6h + L_{effn} \\ W_g = 6h + W_n \end{cases} \qquad (45)$$

3. Result

The characteristics and basic parameter of the RMPA has been calculate and to bring again in to table 1; it has been derived by optimizing the same as above. All the simulations have been carried out using a FEM software: Ansoft HFSS. Proposed RMPA is printed on the FR-4 substrate with relative permittivity of 4.23 and tangent loss of 0.025 @ 850 MHz, thickness of 1,5 mm; the Top and Bottom metallization is copper with conductivity of $5.8*10^7$ S/m and thickness of the 50 micron. The simulation data and measured data has been compare such as fig. 3. As in graph, fig. 3.1a the difference between the simulated and measured Return Loss is 2 MHz; this could depend on the welding of feed, the connector and its mechanical tolerance, which have been neglected in the simulations. Furthermore, the minimum is half about that measured; this is related to the size of ground plane and the mutual distance from patch. The bandwidth measured is greater than that simulated. The graphs also show that the two VSWR at the same frequencies, there are minor losses in care than the other, therefore the Q factor is more lower. The vector and spectrum analyzer, the signal generator are used to identify the impedances and resonant frequencies for RMPA; the anechoic chamber with broadband linear polarized antennas is used too. The result are below from fig. 3.1 to fig. 3.3, while the printed carry out RMPA is illustrated into fig. 3; while fig. 4 illustrates these steps as displayed on the network analyzer before being imported data into HFSS software for comparison with simulated data.

F	L	W	H	t_s	y_{feed}	ε_{eff}
222	86,4	117,10	1,45	35	29,48	4,14

Tab.1 The parameter of RMPA where the length are mm, thickness in micron and frequency in MHz

Fig. 3 Antenna picture made

Fig. 3.1 Compare between the simulated and measured data of electric feature by the RMPA operating in TM_{010} mode: Return Loss, VSWR, Real part and Imaginary part of Impedance.

Fig. 3.2 Directivity gain vs. frequency in phi=180°, theta=50° way and Peak Directivity gain in phi=0°, theta=0° way for RMPA carry out.

Fig. 3.3 Pattern in two main plane (x-z and x-y plane) and 3-d radiation parameters for proposed RMPA as calculated by HFSS: far-field referring to free space.

Fig. 4 Return Loss, VSWR, Impedance displayed on the Vector Network Analyzer.

4. Conclusion

In this paper a Rectangular Microstrip Patch Antenna analysis has been presented. Firstly, the resonant frequency, size and feed location, the parasitic effects (such as fringing, dielectric, ground plane, etc.), far field pattern and fundamental parameter (such as width band, efficiency, quality factor, etc.) are modelled with the above equation and estimated with MATLAB Tool. Secondly the parameter such as: size, the effective resonant frequency, feed point location, etc., has been optimize and the numerical model investigated with FEM electromagnetic simulation technique. Finally the RMPA has designed on a standard FR-4 substrate and can be realized with conventional PCB techniques; after, the basic parameter has been measure (fig. 4.2, 4.3) and compare it with the simulation data. The result to demonstrate that the RMPA model, as well as describing the behaviour of the antenna; it's also provides various real implications in the design itself.

5. References

[1] A. K. Bhattacharjee, S.R. Bhadra, D.R. Pooddar and S.K. Chowdhury, "Equivalence of impedance and radiation properties of square and circular microstrip patch antennas," IEE Proc. Vol.136, no. 4, pp. 338-342, 1989.

[2] E.Chang, S.A. Longand W.F. Richards, "An experimental investigation of electrically thick rectangular microstrip antennas," IEEE Trans. Antennas Propagat., vol. AP-34, no. 6, pp.767-772, 1986.

[3] R.G. Voughan, "Two-port higher mode circular microstrip antennas," IEEE, Trans. Antennas Propagat., vol. 36, no.,3, pp.309-321, 1988.

[4] I.J. Bahl and P. Bhartia, "Microstrip antennas," Artech House, Inc. USA 1980.

[5] D.M.Pozar "Input impedance and mutual coupling of rectangular microstrip antennas," IEEE Trans. Antennas Propagat., vol. AP-30, no. 6, pp. 1191-1196,1982.

[6] Y.T. Lo, D. Solomon and W.F. Richards "Theory and experiment on microstrip antennas," IEEE Trans. Antennas Propagat., vol. AP-27, no. 2, pp 137-145, 1979.

[7] A.G. Derneryd, and A. G. Lind, "Axtended analysis of rectangular microstrip resonator antnnas," IEEE Trans. Antennas Propagat., vol. AP-27, no. 6, pp. 846-849.

[8] J.R. James and P.S. Hall, "Handbook of microstrip antennas," Peter Peregrinus Ltd, London 1989.

[9] G.P. Gauthier and G.M. Rebeiz, "Microstrip antennas on synthesized low dielectric-constant substrate," IEEE Trans. Antennas Propagat., vol. 45, no. 8, pp. 1310-1313,1997.

[10] K.R: Carver and J.W. Mink, "Microstrip antenna technology," IEEE trans., Antennas Propagat., vol. AP-29, no. 1, pp. 2-23, 1981.

[11] P. Hammer, D.V. Bouchante, D. Verschraven, and A.V. Capelle, "A model for calculating the radiation field of microstrip antennas," IEEE Trans. Antennas Propagat., vol. AP-27, no. 2, pp. 267-270, 1979.
[12] K.F. Lee, R.Q. Lee, T. Talty, "Microstrip subarray with coplanar and stacked parastic elements," Electromagnetics letter, vol. 26, no. 10, pp. 668-669, 1990.
[13] C.A. Balanis, "Adavanced Engineering Electromagnetics," Ed. John Wiley & Son, New York, chp. 8, 1989.
[14] C.A. Balanis, "Antenna Theory, Analysis and Design," 2th edition, John Wiley & Son New York, Chpp.12,14, 1997.
[15] J. Huang, "The finite ground plane effect on the Microstrip Antenna radiation pattern," IEEE Trans. Antennas Propagat., vol. AP-31, no. 7, pp. 649-653, july 1983.
[16] Thomas A. Milligan, "Modern Antenna Design," 2th edition, IEEE Interscience Press New York, chpp. 2,6, 2005.
[17] Girish Kumar, K.P. Ray "Broadband Microstrip Antenna" , Artech House